Embedding of the Higson compactification into the product of adelic solenoids

A. Dranishnikov and J. Keesling

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■ DEFINITION. If (X, d) is a metric space, and f : X → ℝ is a bounded continuous function, we say that f is *slowly* oscillating if

 $\lim_{x\to\infty} diam(f(B_r(x))) = 0$

for any fixed *r* and *x* tending to infinity in *X*, i.e. $d(x, x_0) \rightarrow \infty$ for some $x_0 \in X$.

The *Higson compactification* \overline{X} of X is the smallest one containing X densely so that all bounded slowly oscillating functions extend to \overline{X} .

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Let C_h be the set of all bounded slowly oscillating functions. Then the Higson compactification of X can be obtained by taking the closure of the image of the embedding

$$(f)_{f\in C_h}: X \to \prod_{f\in C_h} [inf(f), sup(f)] \cong I^{C_h}.$$

The Higson compactification is similar to the Stone-Čech.
On the other hand, Z \ Z = R \ R but βZ \ Z ≠ βR \ R.

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Image: A matrix

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Higson Conjecture. For the universal covering X of a closed aspherical manifold M given the lifted from M metric satisfies,

 $\check{H}^i(\overline{X}) = 0$ for i > 0.

- A manifold is called *aspherical* if its universal covering is contractible.
- **Example:** *n*-torus is aspherical, the universal covering is \mathbb{R}^n .

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• Assembly map: $\alpha : \mathcal{K}_*(B\pi) \to \mathcal{K}_*(\mathbb{C}_r\pi)$

- Analytic Novikov conjecture: α is a monomorphism.
- **Baum-Connes conjecture**: α is an isomorphism.
- Coarse assembly map: $A: K_*^{\text{lf}}(E\pi) \to K_*(C_{\text{Roe}}^*(E\pi)).$
- Coarse Baum-Connes conjecture: *A* is an isomorphism.
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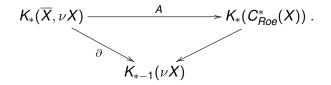
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Coarse assembly map

There is a commutative triangle



where νX is the Higson corona: $\overline{X} = X \cup \nu X$.

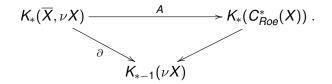
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The exact sequence of the pair $(\overline{X}, \nu X)$ implies that if \overline{X} is acyclic, then ∂ is an isomorphism.

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Theorem. (Keesling) $\check{H}^1(\overline{X}) \neq 0$ for any connected unbounded metric space *X*.

Recall that $\check{H}^1(X) = [X, S^1]$.

For $X = \mathbb{R}$ a nontrivial class is given by $f : \mathbb{R} \to S^1$ with decaying variation, $f(t) = exp(2\pi i\sqrt{t})$. Then $\overline{f} : \overline{\mathbb{R}} \to S^1$ cannot have a lift (\sqrt{t} is unbounded).

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Main Result

Theorem 1.(Dr.-Keesling)

(Dr.-Keesling) Every simply connected proper geodesic metric space X admits an embedding of its Higson compactification into the product of adelic solenoids

$$F: \bar{X} \to \prod_{\mathcal{A}} \Sigma_{\infty}$$

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that induces an isomorphism of 1-dimensional Čech cohomology.

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The *p*-adic solenoid Σ_p is the inverse limit:

$$\Sigma_{p} = \lim_{\leftarrow} \{ S^{1} \stackrel{p}{\leftarrow} S^{1} \stackrel{p}{\leftarrow} S^{1} \leftarrow \cdots \}$$

of unit circles $S^1 \subset \mathbb{C}$ where the bonding maps are $z \mapsto z^p$.

- The universal cover $\mathbb{R} \to S^1$ lifts to an injective group homomorphism $\mathbb{R} \to \Sigma_p$.
- The kernel of $\Sigma_{\rho} \to S^1$ is the group of *p*-adic integers \mathbb{A}_{ρ} .
- Note that $\mathbb{A}_{\rho} \cap \mathbb{R} = \mathbb{Z}$.
- Clearly, $\Sigma_{\rho} = (\mathbb{R} \times \mathbb{A}_{\rho})/\mathbb{Z}$ for the diagonal embedding $\mathbb{Z} \to \mathbb{R} \times \mathbb{A}_{\rho}$.

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The adelic solenoid can be defined as

$$\Sigma_\infty = (\mathbb{R} imes \hat{\mathbb{Z}})/\mathbb{Z}$$

where $\hat{\mathbb{Z}}$ is the profinite completion of the integers and $\mathbb{Z} \to \mathbb{R} \times \hat{\mathbb{Z}}$ is the diagonal map.

All the properties of *p*-adic solenoids hold for Σ_∞.
 Additionally, H¹(Σ_∞; Z_p) = 0 for all *p*.

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Corollary.

For simply connected proper geodesic metric space X,

$$\check{H}^1(\overline{X};\mathbb{Z}_p)=0$$

for all p.

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■ Mod *p* Higson Conjecture. $\check{H}^i(\overline{X}; \mathbb{Z}_p) = 0$ for i > 0 for the universal coverings *X* of aspherical manifolds.

The above Corollary states that the mod 2 Higson conjecture holds true for i = 1.

By a theorem of Calder and Siegel, the mod p Higson conjecture holds for the Stone-Čech compactification.

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Higson conjecture mod p

Theorem (Dr.-Ferry-Weinberger)

The mod 2 Higson conjecture for $X = E\Gamma$ implies the Novikov conjecture for the group Γ .

- Here $E\Gamma$ is the universal covering of the classifying space $B\Gamma = K(\Gamma, 1)$.
- Note that for Γ = π₁(M) in case of an aspherical *n*-manifold M, the space EΓ × ℝ is homeomorphic to ℝⁿ⁺¹.

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Satellite Results

Theorem 2.

For each discrete group Γ with finite complex $B\Gamma$ there is a Γ -equivariant embedding of the Higson compactification of $E\Gamma$ into the product of adelic solenoids

$$F:\overline{E\Gamma}\to\prod\Sigma_\infty$$

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that induces an epimorphism of the integral 1-dimensional Čech cohomology.

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Satellite Results

Theorem 3.

For any p every simply connected proper geodesic metric space X admits an embedding of its Higson compactification into the product of p-adic solenoids

$$F: \bar{X} \to \prod_{\mathcal{A}} \Sigma_{\mathcal{P}}$$

Image: A matrix

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that induces a rational isomorphism of 1-cohomology.

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Essential embedding into the product

- We call a map $f : X \to Z$ essential if every map $g : X \to Z$ homotopic to f is surjective.
- We call a subset $X \subset \prod Z_{\alpha}$ *essential* if its projection on each factor $p_{\alpha} : X \to Z_{\alpha}$ is essential.
- Corollary of Main Resut: For any p every simply connected proper geodesic metric space X admits an essential embedding of its Higson compactification into a product of p-adic solenoids.

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- We define a continuum K_p = Σ_p/ ∼ to be the quotient space under the identification x ∼ −x.
- A theorem of Bellamy says that K₂ is homeomorphic to the Knaster continuum, also known as the Bucket handle continuum.
- Proposition. Any surjective map $f : Y \to K_p$ of a connected compact Hausdorff space is essential.
- Question. Is it true that for any indecomposable continuum X every surjective map $f: Y \rightarrow X$ of a compact connected space is essential?

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Embedding into the product of Knaster continua

Theorem 4.

For any p and any simply connected finite dimensional proper geodesic metric space X its Higson compactification can be essentially embedded into the product of continua K_p .

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THANK YOU!!!

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