Base Tree Phenomenological Horizons

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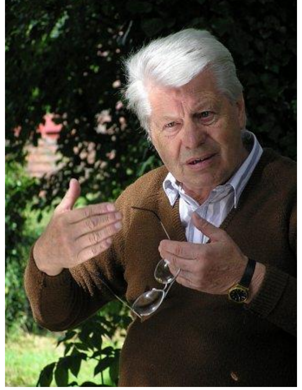
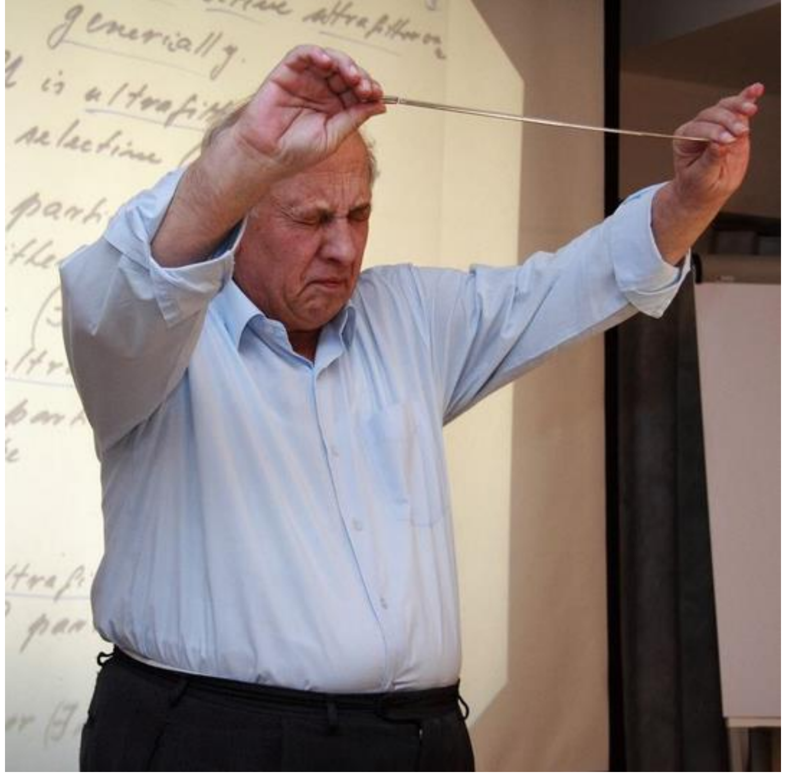
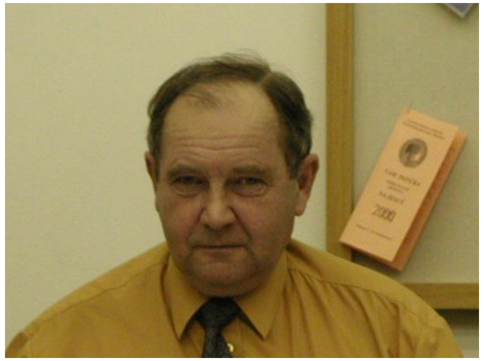
In my first life, upon a time, in the wonderland of set-theoretic topology…

Thanks to





- PhD advisor1 – Petr Vopenka - the man who was my role model – phenomenology seminar
- PhD advisor2 – Bohuslav Balcar – a man who was my mathematical teacher, introduced me to problems, techniques, Prague traditions and contacts abroad
- Lev Bukovsky – his seminar in Kosice was my safe heaven, hideout in uncertain times …
- All my colleagues, students, coauthors, …
- Charles University study of theoretical cybernetics – whole time in Kosice lectured Turing machines, recursive functions, logic programming, … this gave me later foundation and starting point to my second life activities



Outline of this talk

- Sources and Components – motivations, tracks
  - from Balcar Pelant Simon Baire like approach on one side and
  - from Fichtengolz slow/faster converge/diverge series on the other
- How did it evolve (pure topology (co-absoluteness) - series)
- The (ℓ^1 , $\leq^*$) - ($c_0^+ \setminus \ell^1$, $\geq^*$) horizon and $\left(\bigcap \ell^{1+1/k}\right) \setminus \ell^1$ plateau
- RO($c_0^+ \setminus \ell^1$, $\geq^*$) and RO($\wp(\omega) /_{\text{fin}}$, $\subseteq^*$) can be isomorphic
- But need not always in ZFC (Fuchino, Mildenberger, Shelah, V)
- Problems, hypothesis
- Horizon, pass, sensing infinity, infinitesimals, ideological, cultural, technological, … horizons

Motivations

- [BPS] B. Balcar, J. Pelant, P. Simon. The space of ultrafilters on N covered by nowhere dense sets. Fund. Math. 110 (*1980*), 11-24
- G. M. Fichtenholz, The course of differential and integral calculus, Fizmatgiz, Moscow, *1959* (Russian) $\sum_{n=k}^{\infty} a_n = o(\sum_{n=k}^{\infty} b_n), k \to \infty$
- N. N. Kholshchevnikova, Unsolvability of several questions of convergence of series and sequences, Mat. Z. 34 (1983), 711-718 (Russian 1981)
- Toposym 1986 - Set-theoretic characteristic versus gaps in convergence of series and P(ω)/fin
- BELASOVA, J.—EWERT, J.—SALAT, T. : On the effectiveness of tests for the absolute convergence of infinite series, Bull. Math. Soc. Sci. Math. R.S. Roumanie (N. S.) 33 (1989), 3-8.

How did it evolve …

Topology – more general – are spaces co-absolute or not? From [BPS] …

- Broverman-Weiss 82, Wiliams 82, vanMill-Wiliams 83, …
- 89 Dow – tree π-bases $\beta N \setminus N$
- Dordal, Laver, van Douwen, …
- 98 Shelah-Spinas
- 98 Dow $RO(\beta R \setminus R) \neq RO(\beta N \setminus N)$ - very similar to our approach
- 2015 Balcar-Doucha-Hrusak BTP

From analysis, asymptotics, … series, sequences, $\omega^* \setminus Q$,

- 85 Coplakova-V Q-points, 94 ctd., Toposym 96
- 92 V. A note on the effectiveness of tests for the absolute conv. div. of infinite series (Belasova-Ewert-Salat)
- 93 MAx $RO(c_0^+ \setminus \ell^1, \geq^*) = RO(\beta N \setminus N)$
- 95 Krajci-V same for finite partitions
- 99 FMSV $RO(c_0^+ \setminus \ell^1, \geq^*) \neq RO(\beta N \setminus N)$, like 98 Dow, only to keep $\Sigma a_n = \infty$ …

Baire's theorem is about our ability to climb a horizon (speed, distance (method)) – surprise in ω^* - similar to collapsing algebra Coll($\mathfrak{h}$, 2^ω)

- Matrix $\Theta \subseteq \wp$(Open(P))
- Shattering matrix
- Refining matrix
- Base matrix
- Definition [BPS]. Let P be a dense in itself topological space. Define

  κ(P) = min{|Θ|: Θ is a shattering matrix for P}

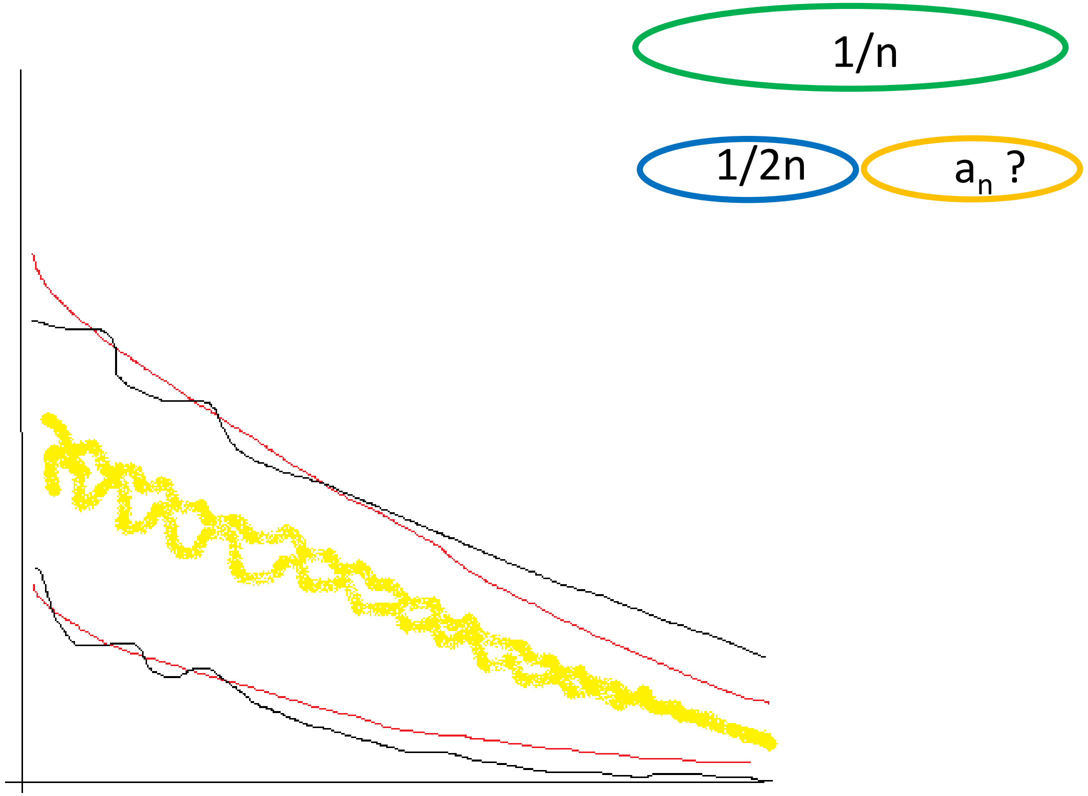
Shift of notation $\kappa(P,\tau) \rightarrow \mathfrak{h}(P,<)$

- The *height* of a partial order (P,≤), $\mathfrak{h}$(P) shortly, is the minimal cardinality of a system of open dense subsets of P such that the intersection of the system is not dense.
- An equivalent definition involves maximal antichains: $\mathfrak{h}$(P) is equal to the minimal cardinality of a system of maximal antichains from *P* that do not have a common refinement.
- **One sided horizon…**

B. Balcar, J. Pelant, P. Simon. The space of ultrafilters on N covered by nowhere dense sets. Fund. Math. 110 (1980), 11-24

B. Balcar, M. Doucha, and M. Hrusak, Base tree property, Order 32 (2015), no. 1, 69–81

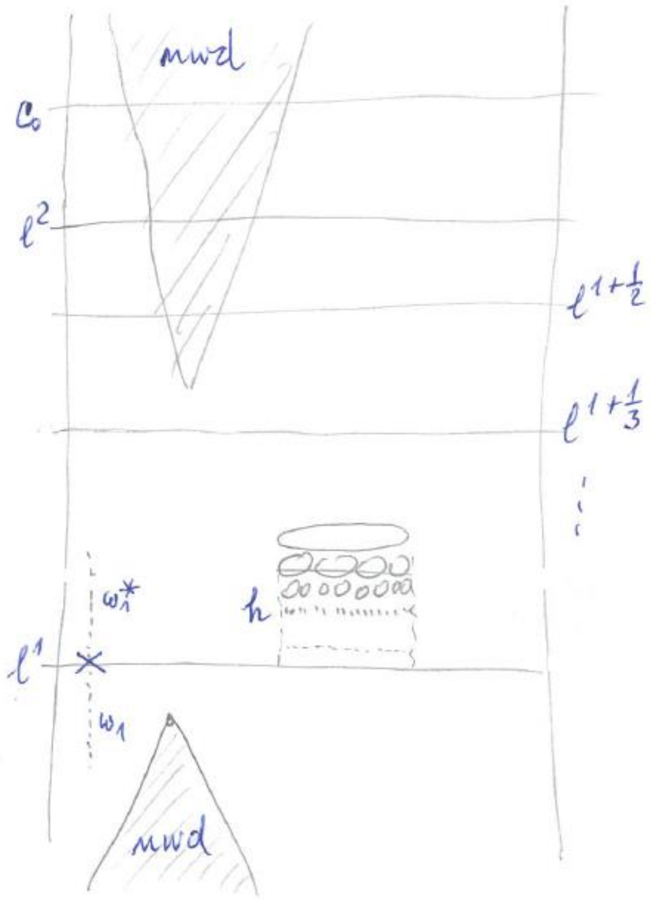
Perception: a (two sided) horizon between convergent and divergent series



- Comparison tests
- What is stronger under eventual dominance $a_n <^* b_n$ ($\leq^*$ resp.)?
  - Divergence: $a_n <^* b_n$ is stronger
  - Convergence: $a_n <^* b_n$ is stronger
- $(\ell^1, \leq^*)$ directed upwards (stronger)
- $(c_0^+ \setminus \ell^1, \geq^*)$ Boolean-like (topology-like) downwards (stronger)
- $(c_0^+ \setminus \ell^1, \geq^*)$ is **not separative**: $1/2n < 1/n$ but **there is no** $a_n \in c_0^+ \setminus \ell^1$, $a_n <^* 1/n$, s.t.

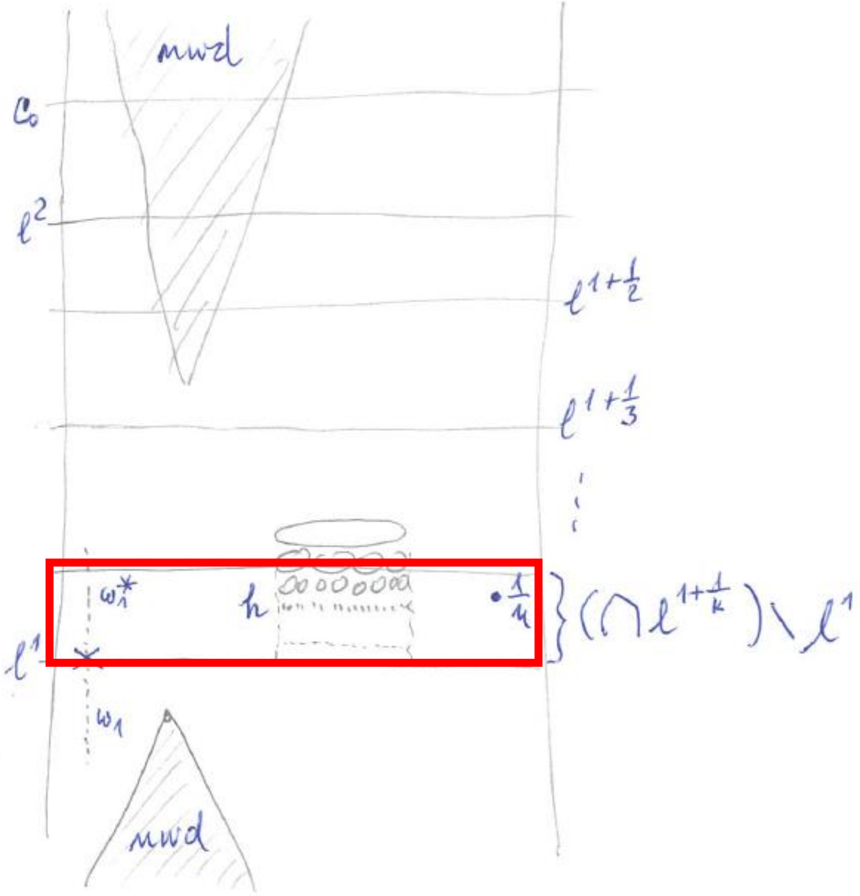
$$\min(a_n, 1/2n) \in \ell^1$$

c_0^+: $a_n \geq 0, \lim a_n = 0, \Sigma a_n = +\infty$ or $\Sigma a_n < +\infty$



- Eventual dominance $a_n <^* b_n$ decides only topologically small sets (horizon is topologically large?)
- $(\ell^1, \leq^*)$ directed upwards, $\mathfrak{b}(\ell^1, \leq^*)$ ZFC sensitive
- $(c_0^+ \setminus \ell^1, \geq^*)$ Boolean downwards $\mathfrak{t}(c_0^+ \setminus \ell^1, \geq^*)$ ZFC sensitive
- There is an (ω_1, ω_1^*) gap (**narrow path**)
- There is base tree (**broad way**)

c_0^+ horizon



- Eventual dominance $a_n <^* b_n$
- $(\ell^1, \leq^*)$ directed upwards, $\mathfrak{b}(\ell^1, \leq^*)$
- $(c_0^+ \setminus \ell^1, \geq^*)$ Boolean downwards
- (ω_1, ω_1^*) gap (narrow path), base tree (broad way)
- **Plateau** $\left(\bigcap \ell^{1+1/k}\right)\setminus\ell^1$ on pass …
- Explicit language of analysis is countable
- Set-theoretic topology can handle this phenomenon

Various $\mathfrak{h}$(P) formulations

The following are equivalent with $\kappa < \mathfrak{h}(P)$

($\approx$ a separative quotient)

(1) RO((P,$\leq$)/$\approx$) is κ-distributive.

(2) The intersection of κ open dense subsets of (P,$\leq$) that are closed under $\approx$ is dense in (P,$\leq$).

(2') The intersection of κ open dense subsets of (P,$\leq$)/$\approx$ is dense in (P,$\leq$)/$\approx$

(3) Every family of maximal κ antichains in P has a refinement.

(3') Every family of maximal κ antichains in P/$\approx$ has a refinement.

(4) Forcing with (P,$\leq$)/$\approx$ does not add a new function from κ to ordinals.

(5) In the following game G(P,κ) the player INC does not have a winning strategy.

The game G(P,κ) is played in κ rounds, and the two players INC and COM choose p_α^{INC}, p_α^{COM} in the αth round such that for all $\alpha < \beta < \kappa$,

$$p_\alpha^{INC} \geq p_\alpha^{COM} \geq p_\beta^{INC} \geq p_\beta^{COM} .$$

In the end, player INC wins iff the sequence of moves does not have a lower bound in P or if at some round he/she has no legal move.

S. Fuchino, H. Mildenberger, S. Shelah, and P. Vojtas, *On absolutely divergent series*, Fund. Math. **160** (1999), no. 3, 255–268

$\mathfrak{h}$(P) – we will use (2)

The following are equivalent with $\kappa < \mathfrak{h}(P)$

($\approx$ a separative quotient)

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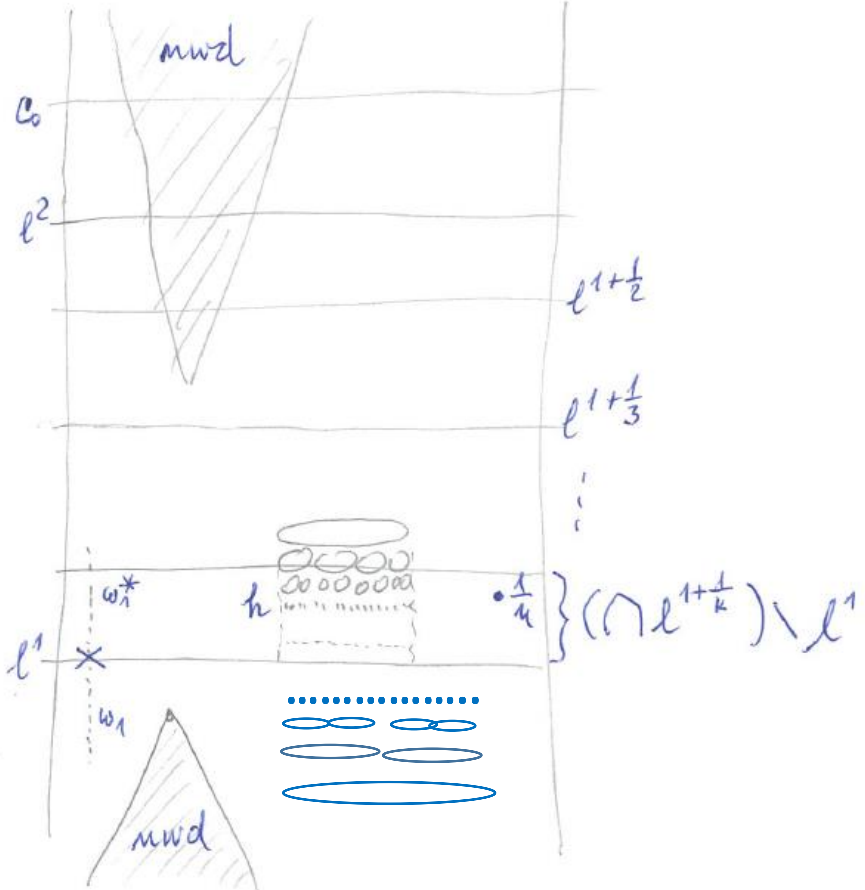
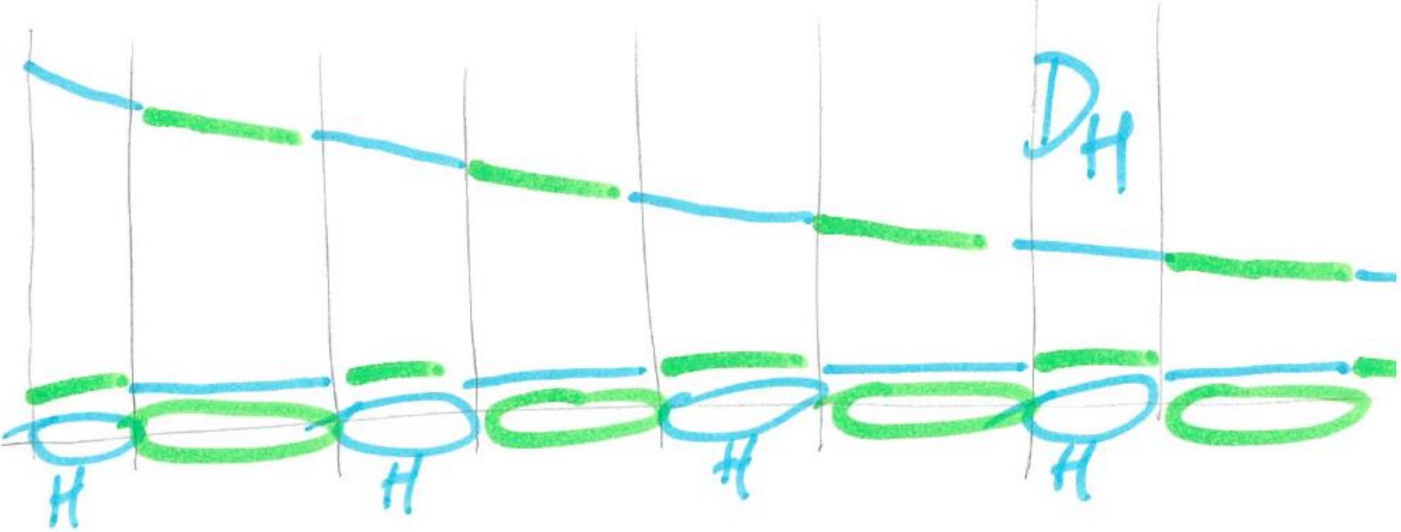
Are Base Tree Phenomenological Horizon always (cBa) isomorph?



- Balcar-Doucha-Hrusak Base Tree Property - BTP
- In ℓ^1 $a_n \trianglelefteq^* b_n$ iff $\frac{\sum a_{n+1}}{\sum a_n} \leq^* \frac{\sum b_{n+1}}{\sum b_n}$ is Boolean upwards and BTP, $\mathfrak{t}(\trianglelefteq^*)$
- $(c_0^+ \setminus \ell^1, \geq^*)$ Boolean downwards BTP
- Under CH **are** all cBA isomorph **– are they always?** PV, PAMS 117,1 (1993, Toposym 1991)
- Presented 1990 to S. Shelah, last correction at **TOPOSYM 96**, Fund.Math. 1999

$$\mathrm{Con}(\, \mathfrak{h}(c_0^+ \setminus \ell^1, \geq^*) < \mathfrak{h}(\wp(\omega) /_{\mathrm{fin}}, \subseteq^*) \,)$$

S. Fuchino, H. Mildenberger, S. Shelah, and P. Vojtas, *On absolutely divergent series*, Fund. Math. **160** (1999), no. 3, 255–268



Construction
very similar to Dow98
only problem is to keep
$\Sigma a_n = \infty$

- *In any extension obtained by the* $\aleph_2$*-stage countable support iteration of Mathias forcing over a model of* CH, *the complete Boolean algebra generated by the* ***separative*** *quotient of absolutely divergent series under eventual dominance is not isomorphic to the completion of* $\wp(\omega)/_{\text{fin}}$

Suppose that $\bar{b} \in (c_0 \setminus \ell^1)^{V[G]}$. There is some $\delta < \omega_2$ such that $\bar{b} \in V[G_\delta]$. We choose a family $\langle D_\nu \mid \nu \in \omega_1 \rangle \in V[G]$ such that $\langle D_\nu \mid \nu \in \omega_1 \rangle$ is an enumeration of

$$(4.1)\quad \left\{ \left\{ \bar{a} \in (c_0 \setminus \ell^1)^{V[G]} \;\middle|\; \sum_{l \in H} a_l < \infty \text{ or } \sum_{l \in \omega \setminus H} a_l < \infty \right\} \;\middle|\; H \in ([\omega]^\omega)^{V[G_\delta]} \right\} = D_H$$

Claim. Intersection of D_ν is not dense below b in V[G].

S. Fuchino, H. Mildenberger, S. Shelah, and P. Vojtas, *On absolutely divergent series*, Fund. Math. **160** (1999), no. 3, 255–268

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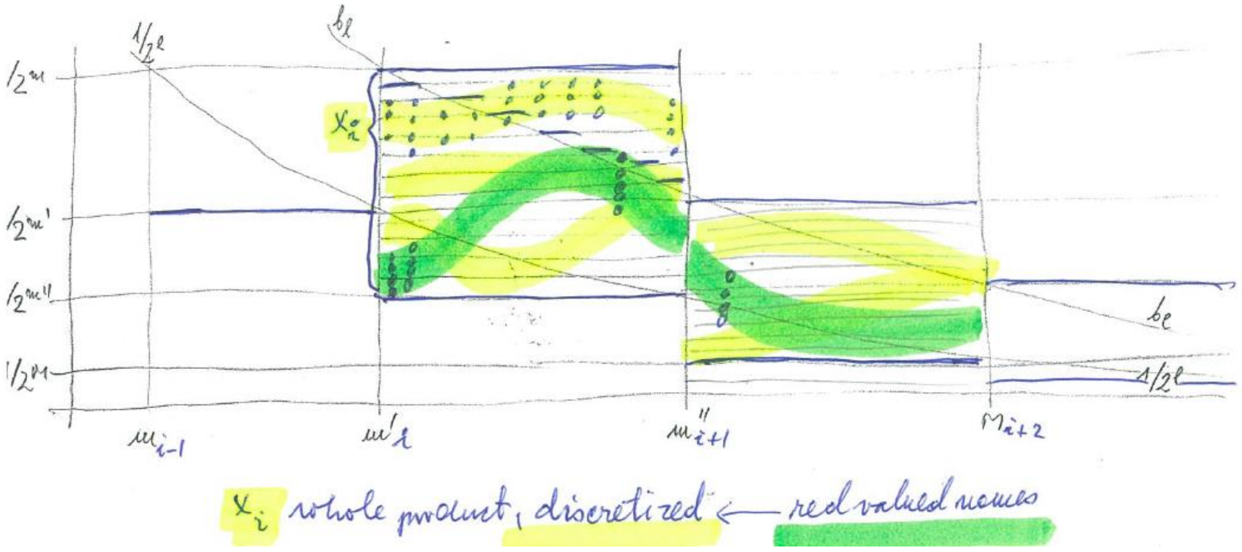
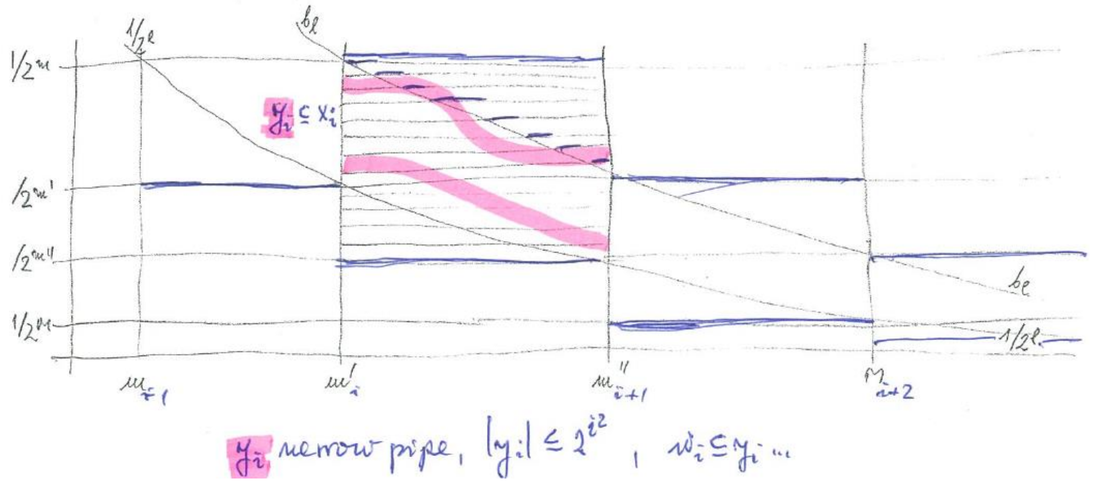
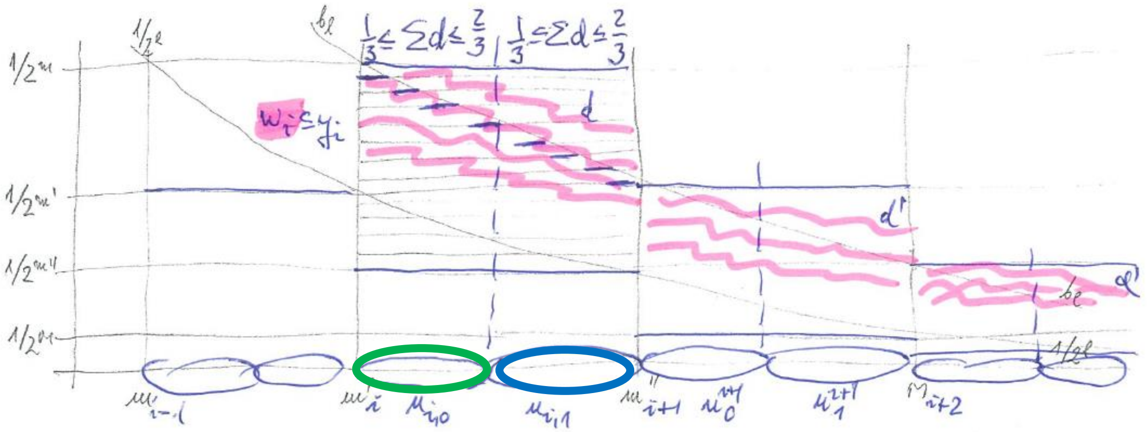
Fund. Math. **160** (1999), no. 3, 255–268



- Proof by contradiction
- Assume D_ν is dense below b (in V[G])
- Let $c \leq^* b$, $c \in \bigcap D_\nu$ in V[G]
- Working in $V[G_\delta]$
- There is a name c' for c in $V[G_\delta]$
- Using $b \in V[G_\delta]$, define m_i^b …
- Discretized name c'' for c in each [m', m''] …

Shelah: Let’s go to casino …

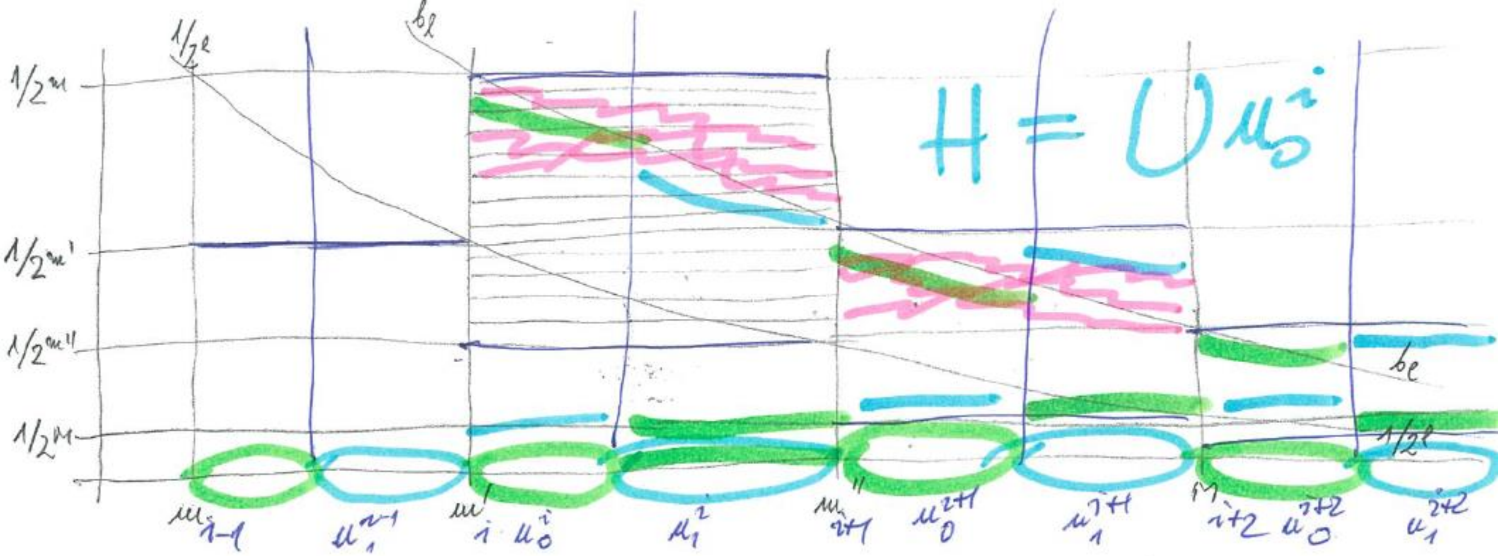




- Discretized name for c’’ in each [m’, m’’]
- Laver property – name c* for c in a **narrow pipe** in each y_i
- $w_i = \left\{ d \in y_i : \sum_{l=m_i}^{m_{i+1}} d_l > {}^{1}/_{i^2} \right\}$ **here we care about divergence, Dow98 need not to**
- $e \in y_i \rightarrow (\exists^\infty i)(e | [m_i, m_i^{+1}) \in w_i)$
- flip a fair coin to divide [m’, m’’] = $u_0 \cup u_1$, (**Alon-Spencer-Erdös trick**) estimate chance that a $d \in w_i$, h=0,1

$$\frac{1}{3} \leq \frac{\sum\{d_l \mid l \in u_h\}}{\sum\{d_l \mid l \in [m', m'')\}} \leq \frac{2}{3}$$

It is nonzero (large product, narrow pipe, … + some more conditions on m_i)



J’ is the bad guy

With a real parameter in $V[G_\delta]$ (namely $\langle u_{0,i} \mid i \in \omega \setminus \{0\}\rangle$) we define the set

$$J = \{\bar{d} \in (c_0 \setminus \ell^1)^{V[G]} \mid \exists h \in \{0,1\}\ \forall^\infty i \in \omega \setminus \{0\}\ (\bar{d} \restriction u_{h,i} \equiv 0)\}.$$

J is obviously open in $(c_0 \setminus \ell^1, \leq^*)$.

The closure of J under $\approx$ is

J’ is one of D_H

$$J' = \{\bar{d} \mid \exists \bar{d'} \in J\ \forall \bar{e} \leq^* \bar{d}\ (\bar{e} \not\perp \bar{d'})\} \tag{4.15}$$
$$= \left\{\bar{d} \,\middle|\, \exists h \left(\sum_{i \in \omega \setminus \{0\}} \sum_{l \in u_{h,i}} d_l < \infty \right)\right\}.$$

- and c* $\notin$ J’ □

- **Problem.** Does ZFC decide $\mathfrak{h}(c_0^+ \setminus \ell^1, \leq^*) \leq \mathfrak{h}(\wp(\omega)/_{\text{fin}}, \subseteq^*)$?

Problems, hypothesis

- Dow 98 – RO(R*) has a certain two dimensionality – we say a many valued object of investigation – Conjecture:
  - $\mathfrak{h}(R^*) \leq \mathfrak{h}(N^* \times N^*)$
  - $\mathfrak{h}(R^*) \leq \mathfrak{h}(R^* \times R^*)$
- We can repeat this by asking $\mathfrak{h}(c_0^+ \setminus \ell^1, \geq^*) \leq \mathfrak{h}(N^* \times N^*)$? Rephrasing Balcar-Hrusak: Is $\mathfrak{h}(c_0^+ \setminus \ell^1, \geq^*) \leq \min(\mathfrak{h}, \mathrm{add}(\mathcal{M}))$?
- Is it ZFC consistent $RO(\ell^1, \trianglelefteq^*) \not\cong RO(\wp(\omega)/_{fin}, \subseteq^*)$? (Shelah: "dirty computing" …)
- (with S. Krajci) $\mathbb{P}$ = {partitions $P \subseteq [\omega]^{<\omega}$ of ω s.t. $\limsup p_n = +\infty$ }, another model of approaching infinity

  $P \Subset Q$ if $(\forall p \in P)(\exists!\, q \in Q)(p \subseteq q)$

  $(\mathbb{P}, \Subset)$ has BTP, hence under CH isomorphic to all BTP structures **Problem**. Is Con($\mathfrak{h}(\mathbb{P}, \Subset) < \mathfrak{h}(\wp(\omega)/_{fin}, \subseteq^*)$) ? Probably not, it is not a many valued structure

S. Krajči; P. Vojtáš. On the Boolean structure generated by Q-points of ω∗ Acta Univ. Carolin. Math.Phys. 36,2 (1995) 33--38

Problems, hypothesis ctd.

- Fichtengolz like horizon $\sum_{n=k}^{\infty} a_n = o(\sum_{n=k}^{\infty} b_n), k \to \infty$, both in $c_0^+ \setminus \ell^1$ and ℓ^1
- Flaskova / Blobner ZFC $\vdash (\exists \mathcal{U} \in \omega^*)(\forall$1-1 f : N → N$)(\exists U \in \mathcal{U})(f[U] \in \mathcal{I}(1/n))$, i.e. there is a point behind the horizon (Analogy of covering non Q points, these can be called "harmonic points (exist in ZFC)", Gryzlov in ZFC Asymp.density 0-points)
- What are interesting horizon in ω^*? In Katetov ordering? Rudin-Frolik order? Each point $\mathcal{U} \in \omega^*$ is a $\mathfrak{t}$–point, order witnesses of $\mathfrak{t}$-pointedness by $\subseteq$, … more examples in Balcar – Doucha – Hrusak in Order 2015 Base tree property
- Many horizons between small/big, slow/fast (ideal/filter), asymptotic create horizons (e.g. polynomial/exponential, degrees of computability, P/NP,)
- Horizons – one/two sided – Baire/natural – two valued/many valued – narrow/broad
- Is there a border between $(\ell^1, \leq^*)$ and $(c_0^+ \setminus \ell^1, \geq^*)$? **Hyperreals - is there a "boundary" between convergent and divergent series?**

Horizon, pass, sensing infinity, infinitesimals,… ideological, cultural, technological, … horizons

Railway sleepers, last railway sleeper before horizon? Do rails continue behind horizon? Telescope sees further/details.

Blue ridge mountains. What is on the other side? Is there anything?

Approaching a horizon – speed, distance (method)

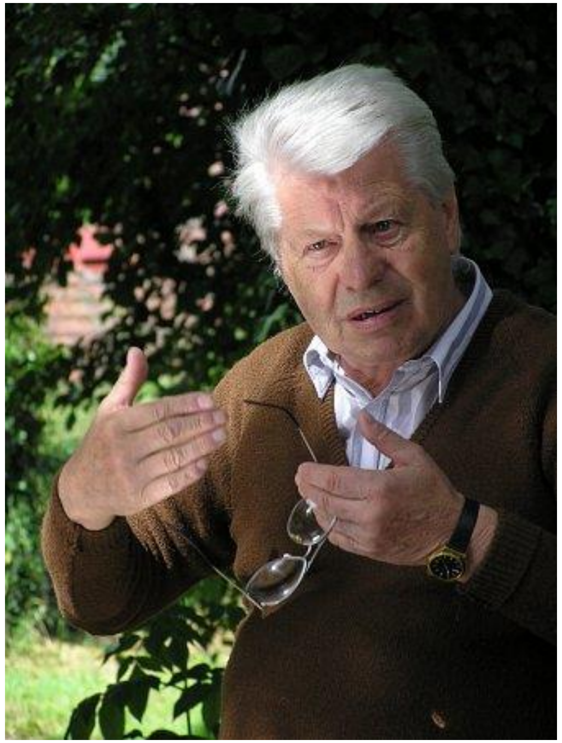
Narrow path / climb the hill / consistency?

Pass at the horizon connects worlds.

Broad way … Can we meet in the pass?

How fast am I climbing

Philosophical/mathematical horizons



- Edmund Husserl
  - – phenomenon of horizons
- H. Jerome Keisler

  - infinitesimals, extension / transfer axiom,
- Petr Vopěnka

  -semisets, prolongation axiom
- Topological boundary, other mathematical horizons …

Real world horizons (Platonist, Aristotelian, physics, phenomenologist, constructivist, …)


Black Hole in space and Point of singularity

- Are all (nontrivial, mathematical) horizons -
  - Either directed
    - Like ($\omega^\omega, \leq^*$)? How many types of such horizons are there? Is there some **spectral theorem?**
    - or trivial, countable, …
  - Or Boolean/topological
    - either ($c_0^+ \setminus \ell^1, \leq^*$) like (many valued)?
    - or ($\wp(\omega)$ /$_{fin}$, $\subseteq^*$) like (two valued)?
  - one/both sided? Baire/natural? two valued/many valued? narrow/broad? **other?**
- Physics - A cosmological *horizon* is a measure of the distance from which one could possibly **retrieve information (Google horizon)** - Particle horizon, Hubble horizon, Event horizon, Future horizon, optical horizon, neutrino horizon, gravitational wave horizon, …

Real world horizons ctd.

- Google horizon, AI, (spiritual) ecology, sustainable…
- The ultimate event horizon/pass (Vopěnka is behind, some of us approaching closer/faster some slower…)

- Cultural horizon, beyond human (man, woman) comprehension (sensing colors), horizons in history – Silk road horizons …

Thank you!

Questions? Comments?

Hated set-theory? Not at all, just pivoted to human behavior experiments … computer science is about human users

"I'm not leaving you. I'm pivoting to another man."

Plato's nature of mathematics

Nature of physics – all particles of same sort behave same

Nature of humans (are not particles)
- Behavior
- Recommendation
- Challenge
- Model
- Method - prototype
- Data
- Metrics
- Experiments
- contribution

… 2015 started to attend B. Balcar's seminar again

Google Scholar

A many valued world with Scott topology
Automatic Tarski lattice continuity
Left lower semicontinuous connectives

Title 1–20	Cited by	Year
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Dependences between definitions of finiteness L Spišiak, P Vojtáš Czechoslovak Mathematical Journal 38 (3), 389-397	10	1988

My personal Toposym history – towards “second life”

- **1974** Theoretical Cybernetics, **1976-80** PhD study under supervision of B. Balcar, **1978 -91** lecturing – logic programming, Turing Machines,

- Toposym 1981 - A transfinite Boolean game and a generalization of Kripke's embedding theorem.

  - Martial law Poland December 13, **1981** to July 22, 1983

- Toposym 1986 - Set-theoretic characteristic versus **gaps** in convergence of **series** and P(ω)/fin.

  - **17.11.1989** political changes in Czechoslovakia, **1990-91** AvHumbold fellow – meeting **S. Shelah** in Halle G. Cantor set theory seminar – presented the problem of isomorphism of RO(divSeries) and RO(P(ω)/fin), cooperation with **S. Fuchino** on subject, **1991** Ramat Gan winter school on Set theory

- Toposym 1991 - Boolean isomorphism between partial orderings of convergent and divergent **series** and infinite subsets of N

  - **1992-98** Extension of computer science (PhD) studies UPJS, **1995** Logic Colloquium Haifa, **1995** Scientific activities in Computer Science – first paper P. Vojtas, L. Paulik: Logic Programming in RPL and RQL. SOFSEM 1995

- Toposym 1996 - On ultrafilters on ω*.

  - **Toposym 1996** Last meeting with **Saharon Shelah** on subject, continuation with **H. Mildenberger,** published **1999** as FMShV:593, Continuing in Computer Science users’ preference learning