Continuous Neighborhoods in Products

Alejandro Illanes

Universidad Nacional Autónoma de México

Prague, July, 2016

A **continuum** is a nonempty compact connected metric space.

For continua X and Y, let π_X and π_Y denote the respective projections onto X and Y.

The product X x Y has the full projection implies small connected neighborhoods (fupcon) property, if for each subcontinuum M of X x Y such that $\pi_X(M) = X$ and $\pi_Y(M) = Y$ and for each open subset U of X x Y containing M, there is a connected open subset of X x Y such that $M \subset V \subset U$.

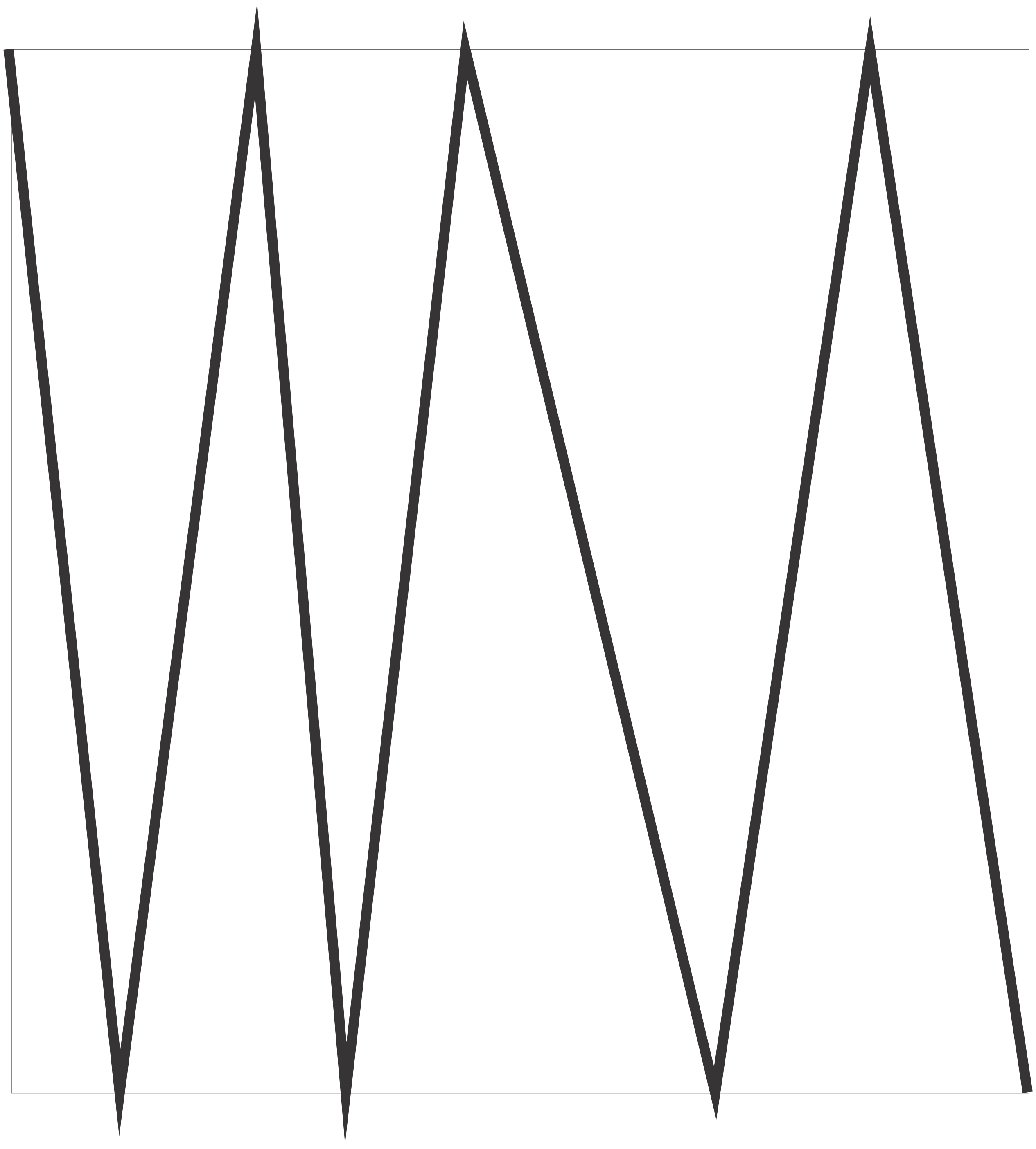
$\pi_X(M) = X$ and $\pi_Y(M) = Y$ and $M \subset U \Rightarrow$ there is open connected V such that $M \subset V \subset U$.

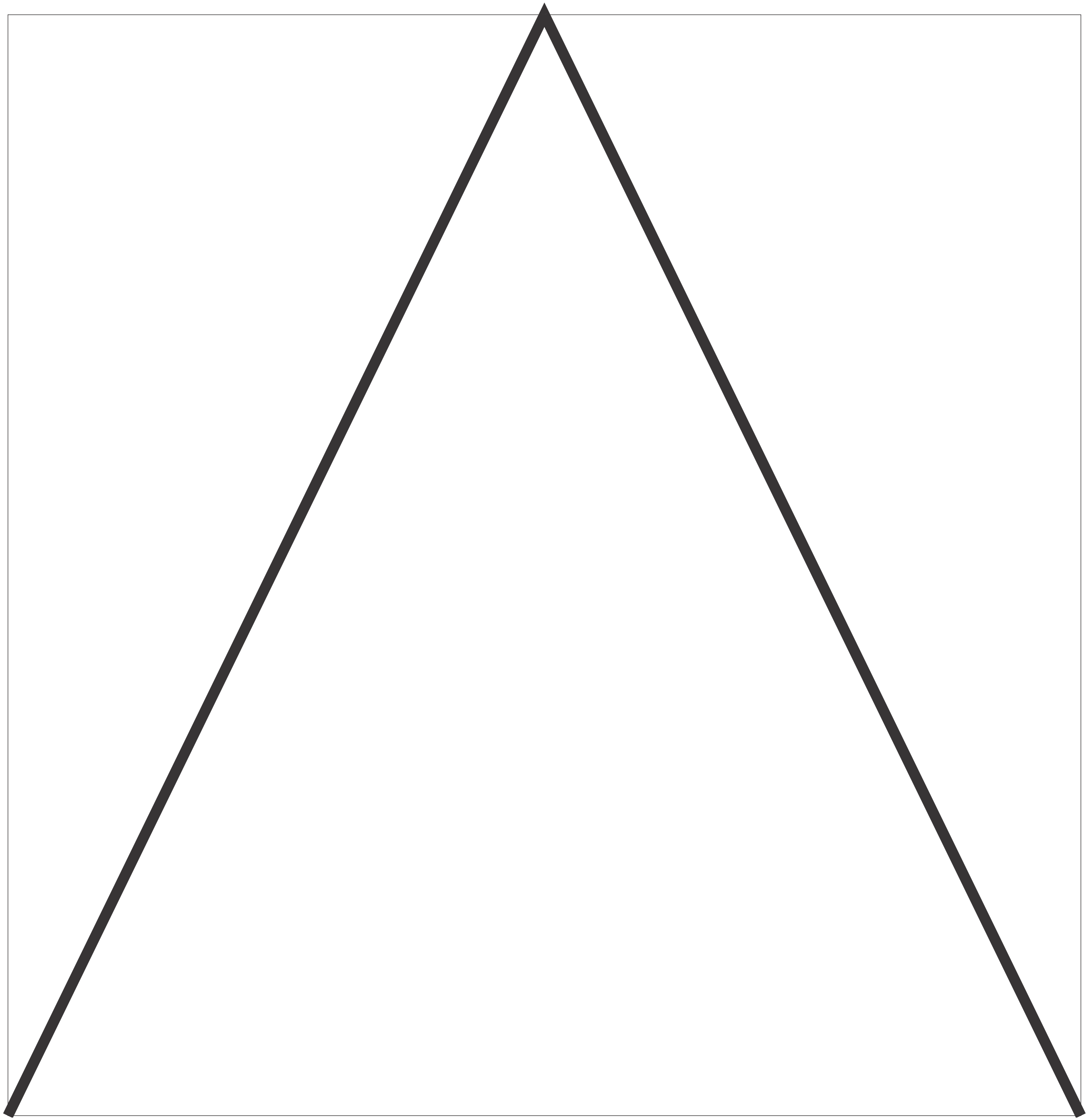
PROP. If X and Y are locally connected, then X x Y has the fupcon property.

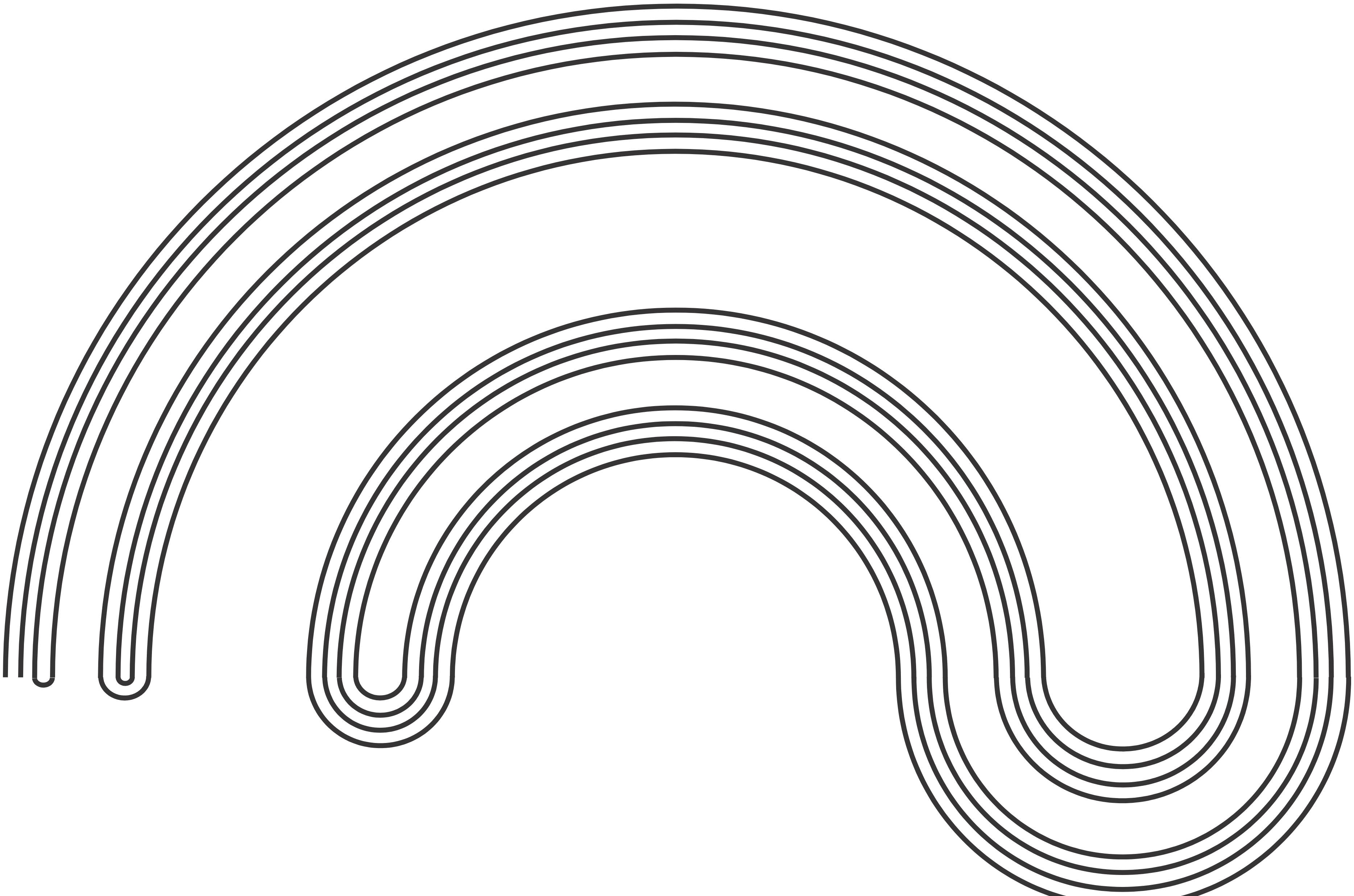
PROP. If M is a subcontinuum of X x Y and M has small connected neighborhoods, then the hyperspace of subcontinua, C(X x Y) of X x Y is connected im kleinen at M.

PROBLEM. Find conditions on continua X and Y in such a way that X x Y has property fupcon.

A **Knaster continuum** is a continuum X which is an inverse limit of open mappings from [0,1] onto [0,1].







THEOREM (D. P. Bellamy and J. M. Lysko, 2014)**.** If X and Y are Knaster continua, then X x Y has fupcon property.

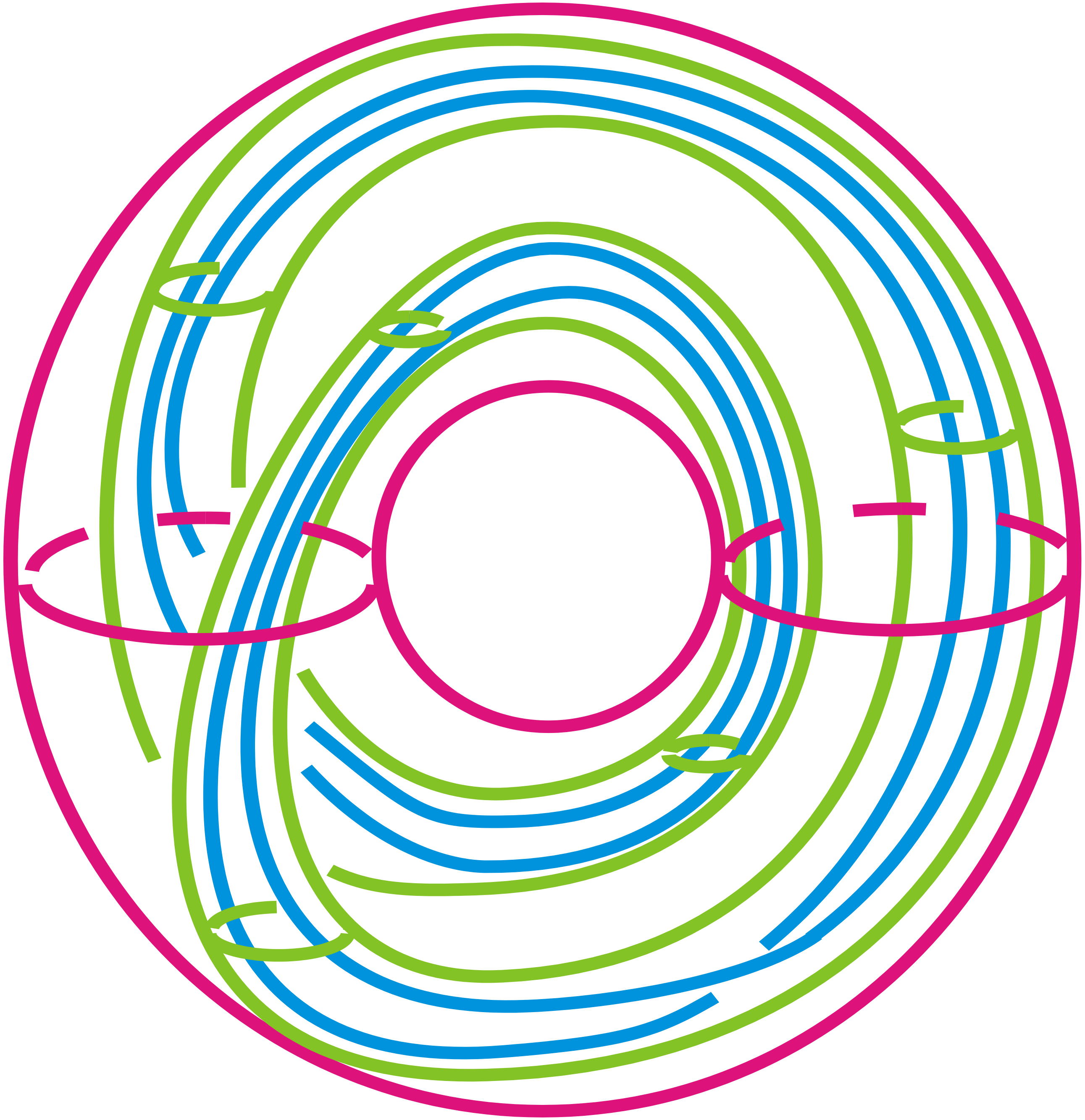
The **pseudo-arc** is any chainable hereditarily indecomposable continuum.

THEOREM (D. P. Bellamy and J. M. Lysko, 2014)**.** If X and Y are pseudo-arcs, then X x Y has fupcon property.

The **n-solenoid, S_n** is the inverse limit of the unit circle in the plane with the mapping $z \to z^n$

THEOREM (D. P. Bellamy and J. M. Lysko, 2014)**.** S_n x S_n does not have fupcon property.

PROBLEM (D. P. Bellamy and J. M. Lysko, 2014)**.** Suppose that (n,m) = 1. Does S_n x S_m have fupcon property?



THEOREM (J. Prajs, 2007). Every pair of subcontinua with nonempty interior of S_n x S_n intersect.

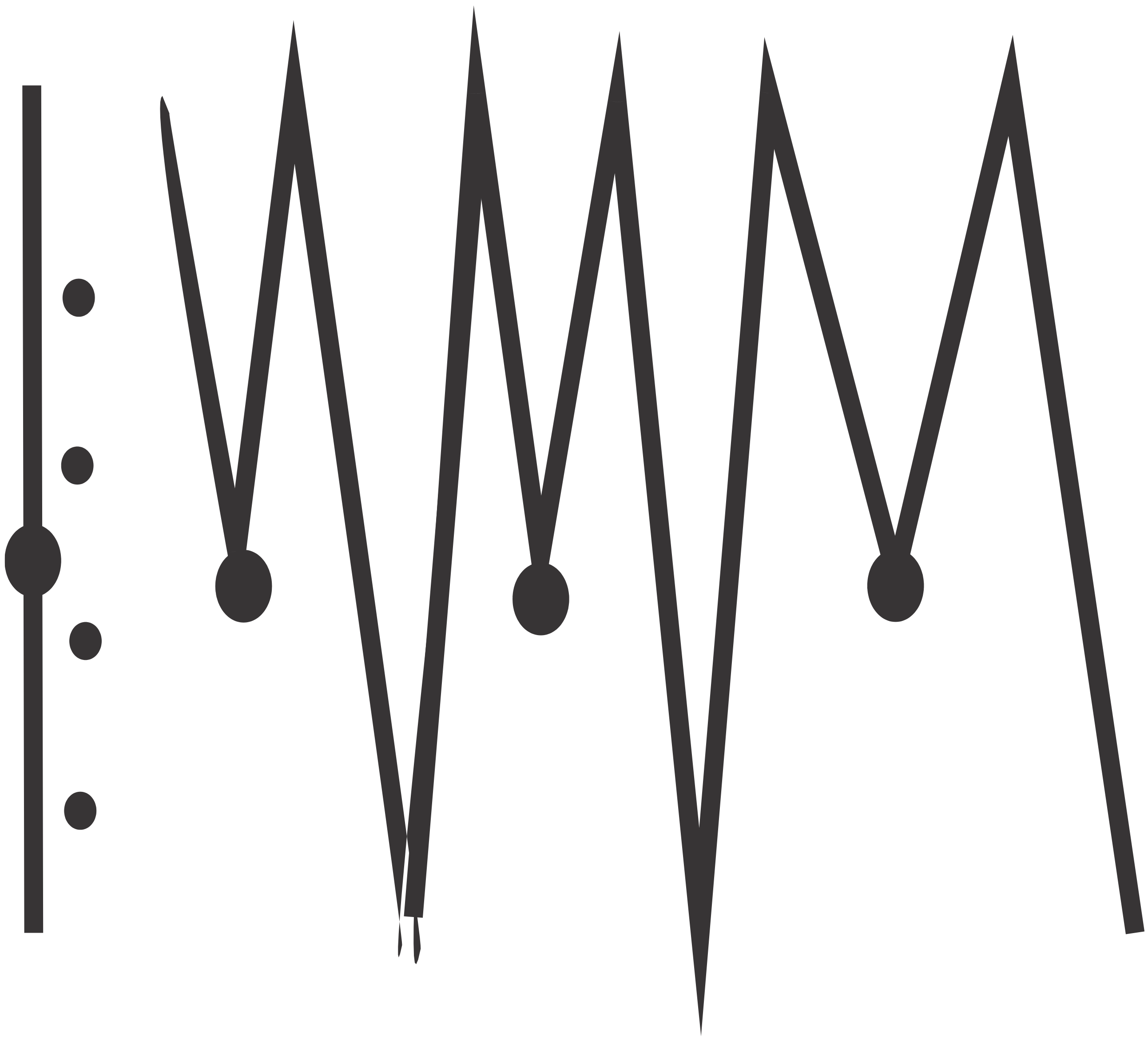
THEOREM (A. I., 1998). If (n,m) = 1, then for each pair of distinct points of S_n x S_m there exist disjoint subcontinua containing them in the respective interior.

THEOREM (A. I., 2015). If X is the pseudo-arc and Y is a Knaster continuum, then X x Y has property fupcon.

PROBLEM. (D. P. Bellamy and J. M. Lysko, 2014)**.** Does the product of two chainable continua have fupcon property?

A continuum X is a **Kelley continuum**, if the following implication holds:

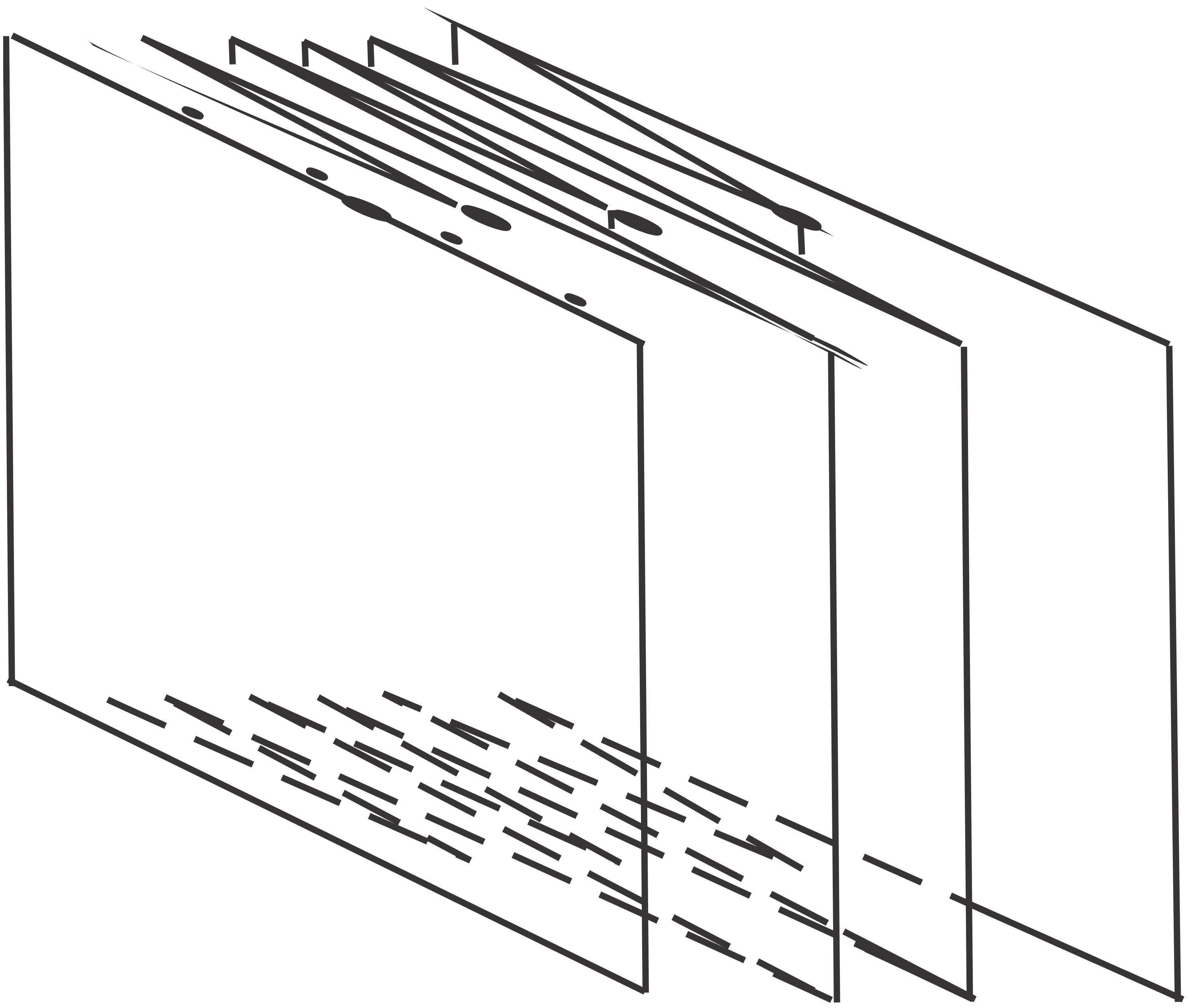
If A is a subcontinuum of X, $p \in A$ and $\lim_{n \to \infty} p_n = p$, then there is a sequence of subcontinua A_n of X such that for all n, $p_n \in A_n$ and $\lim_{n \to \infty} A_n = A$.

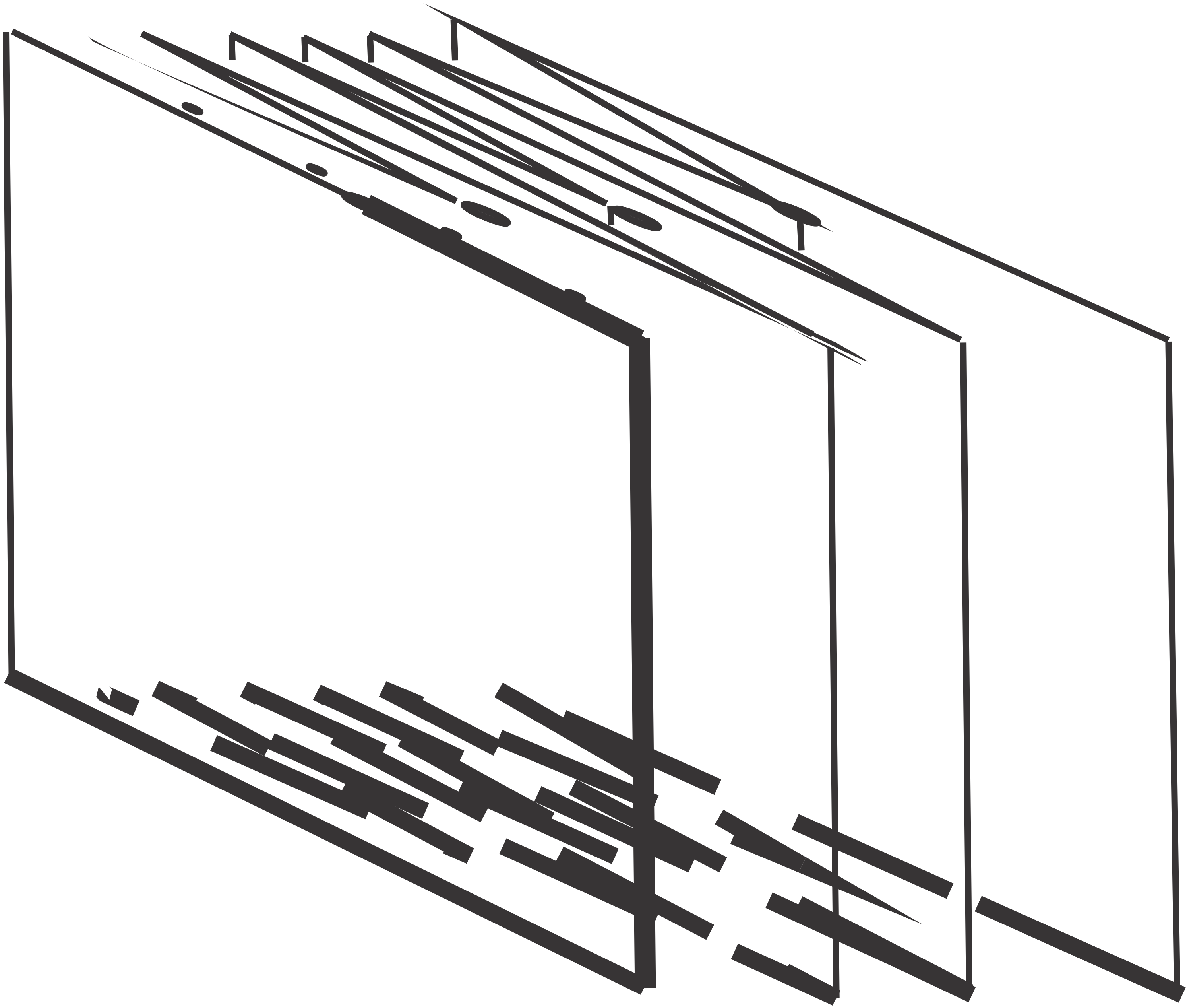


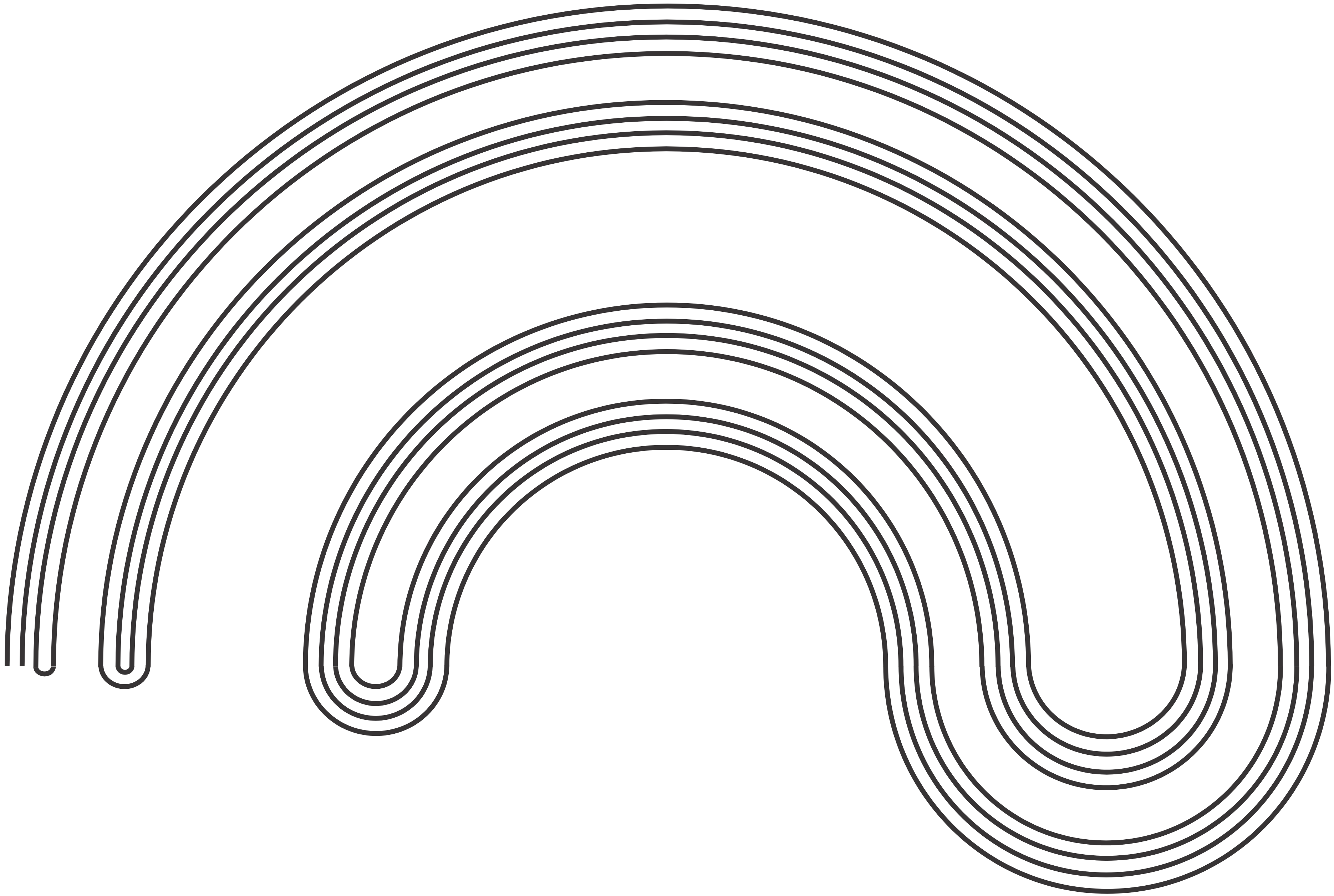
THEOREM (A. I., 2015)**.** if X and Y are continua and X x Y has fupcon property, then X and Y are Kelley continua.

The converse is not true,

EXAMPLE: S_n x S_n







THEOREM (A. I., J. Martinez, E. Velasco, K. Villarreal, 2016)**.** if Y is a Knaster continuum, then S_n x Y has fupcon property.

A **dendroid** is a hereditarily unicoherent arcwise connected continuum.

THEOREM (A. I., J. Martinez, E. Velasco, K. Villarreal, 2016)**.** If X is a dendroid such that X is a Kelley continuum, then X x [0,1] has fupcon property.

THEOREM (A. I., J. Martinez, E. Velasco, K. Villarreal, 2016)**.** if X and Y are chainable continua and they are Kelley continua, then X x Y has fupcon property.

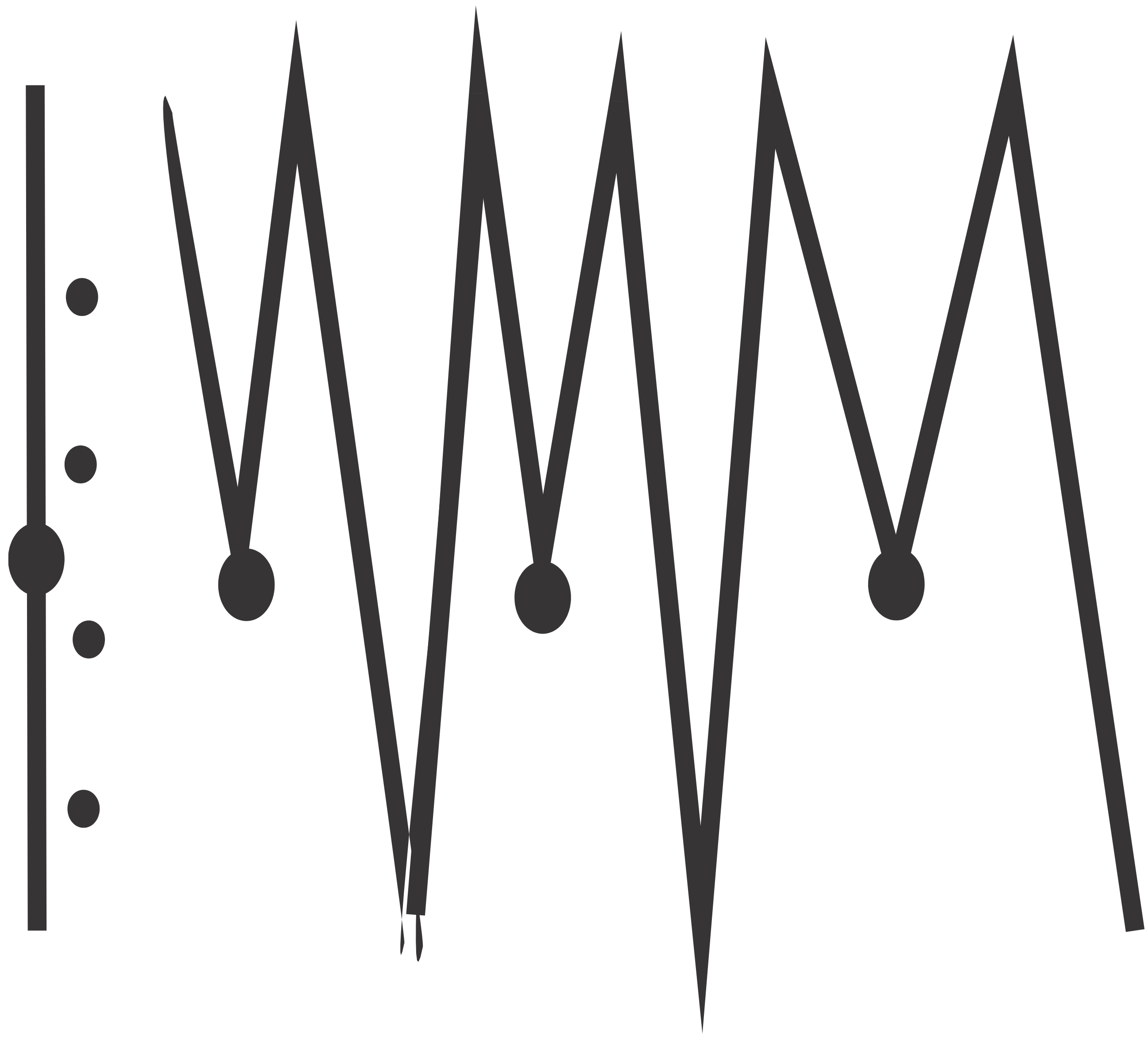
EXAMPLE (A. I., J. Martinez, E. Velasco, K. Villarreal, 2016)**.** There is a Kelley continuum X such that X x [0,1] does not have fupcon property.

For a continuum X, let

$$\Delta_X = \{(x,x) \in X \times X : x \in X\}$$

A continuum X has the **diagonal has small connected neighborhoods property** (diagcon) if for each open subset U of X x X containing Δ_X, there is a connected open subset of X x X such that
$\Delta_X \subset V \subset U$.

D. P. Bellamy asked if each chainable continuum has the diagcon property.



A proper subcontinuum K of a continuum X is an **R_3-continuum** if there exist an open subset U of X and two sequences, $\{A_n\}_{n \in N}$ and $\{B_n\}_{n \in N}$, of components of U such that

$$\lim_{n\to\infty} A_n \cap \lim_{n\to\infty} B_n = K.$$

THEOREM (A. I., 2016)**.** If a continuum X contains an R_3-continuum, then X does not have the diagcon property.

EXAMPLE. S_2 does not have the diagcon property and S_2 does not contain R_3-continua.

THEOREM (A. I., 2016)**.** A chainable continuum X has the diagcon property if and only if X does not contain R_3-continua.

THANKS