An embedded circle into $\mathbb{R}^3$ might not be able to escape before an isotoped linked circle

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The mathematically precise statement of the problem that was intuitively described in the title is following isotopy-extension problem: Given two linked embedded circles and an isotopy for one of them, is it possible to extend the embedding of the second circle to an isotopy, so that at any time of the isotopy both circles remain disjoint, in particular in the case, where the second circle is just parallel to a meridinal curve to the first one? – Since, when isotopying a circle, it cannot bump into itself or just shrink via very small circles down to a point, the result as described in the title is a bit counter-intuitive. However in the first half of the talk a corresponding example will be constructed, based on a construction trick that has already been used to construct Alexander's horned sphere. In the second half of this talk I want to introduce the problem that made me ask this isotopy-extension question: It is the problem of deciding, whether there exist knots (and it is clear that at most totally wild knots might have such a property) that even with respect to ordinary (not necessarily ambient) isotopy are non-equivalent to the trivial knot. In particular the consequences that the newly discovered example has for attacking this problem shall be discussed.