Orderable groups and semigroup compactifications

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This project is dedicated to Eli Glasner on the occasion of his 75th birthday. Our aim is to find some new links between linear (circular) orderability of groups and topological dynamics. We suggest natural analogs of the concept of algebraic orderability for *topological* groups involving order-preserving actions on compact spaces and the corresponding enveloping semigroups in the sense of R. Ellis.

This approach leads to several natural questions. Some of them might be useful also for discrete (countable) orderable groups.

We study the following questions:

Question. Which topological groups can be embedded into the topological group $H_+(K)$ of all circular (linear) order-preserving homeomorphisms of K, endowed with compact-open topology, for some circularly (resp., linearly) ordered compact space K.

Question. Which topological groups G admit proper linearly (circularly) order compact right topological semigroup compactification $G \hookrightarrow S$? When such S is: a) metrizable? b) hereditarily separable? c) first countable?

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