Arbitrarily large countably compact free Abelian groups

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A result of A. Tomita shows that a nontrivial free Abelian group does not admit a group topology whose countable power is countably compact.

In our work, we show that if there are \mathfrak{c} incomparable selective ultrafilters, then, for every infinite cardinal κ such that $\kappa^{\omega} = \kappa$, there exists a group topology on the free Abelian group of cardinality κ without nontrivial convergent sequences and such that every finite power is countably compact.

This answers a question of Dikranjan and Shakhmatov that was posed in a survey by Comfort, Hoffman and Remus.

 $^{^1\}mathrm{The}$ first author received financial support from FAPESP 2017/15709-6. $^2\mathrm{The}$ third author received financial support from FAPESP 2017/15502-2. $^3\mathrm{The}$ fourth author received financial support from FAPESP 2016/26216-8.

