Tame locally convex spaces

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Motivated by Rosenthal's famous l^1-dichotomy in Banach spaces, Haydon's theorem, and additionally by recent works on tame dynamical systems, we introduce the class of *tame* locally convex spaces. This is a natural locally convex analogue of *Rosenthal* Banach spaces (for which any bounded sequence contains a weak Cauchy subsequence). Our approach is based on a bornology of *tame* subsets which in turn is closely related to eventual fragmentability, and some purely topological ideas. This leads, among others, to a generalization of Haydon's theorem for locally convex spaces, a version of Rosenthal's dichotomy strengthening a result of W.M. Ruess, and an extension of the Davis–Figiel–Johnson–Pelczyński (DFJP) factorization technique. It also relates to recent papers by A. Leiderman and V. Uspenski [3], and S. Gabriyelyan and T. Banakh [1]. This project is based on a submitted joint work [2] with M. Megrelishvili.

[1] T. Banakh and S. Gabriyelyan, *Free locally convex spaces*. To appear in FILOMAT.

[2] M. Komisarchik and M. Megrelishvili, *Tameness and rosenthal type locally convex spaces*, 2022. arXiv:2203.02368.

[3] A. Leiderman and V. Uspenskij, *Is the free locally convex space $L(X)$ nuclear?*, 2021. arXiv:2106.13413.