On σ-metacompact function spaces

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We introduce the following property of a family $\mathcal{L}$ of subsets of a set S:

($*$) For all $N \in \mathcal{L}$ and $x \in S$, there exists a finite subset A of N such that, for each $L \in \mathcal{L}$, if $x \in L$ and $L \cap N \neq \emptyset$, then $L \cap A \neq \emptyset$.

We consider compact spaces which have a k-network with property ($*$). Examples of spaces which do not admit a k-network with ($*$) include $\beta\omega$, a compact scattered space of height $\omega + 1$ and the one-point compactification of a tree-space.

Theorem. *If K is a compact space which has a k-network with property ($*$), then $C_p(K)$ is hereditarily σ-metacompact.*

Supercompact spaces are usually defined by the existence of a "binary" subbase for the closed subsets, but according to a known and easy result, every supercompact space has a binary closed k-network.

Proposition. *A family $\mathcal{L}$ of compact closed subsets of a space X is binary if, and only if, for all $N \in \mathcal{L}$ and $x \in X$, there exists $a \in N$ such that, for each $L \in \mathcal{L}$, if $x \in L$ and $L \cap N \neq \emptyset$, then $a \in L$.*

Hence every supercompact space has a k-network with ($*$).

Corollary. *$C_p(K)$ is hereditarily σ-metacompact for every supercompact space K.*

Corollary. *$C_p(K)$ is hereditarily σ-metacompact for every dyadic space K.*