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Preface

The first Topological Symposium took place in 1961 at a time when the world was still painfully divided during the Cold War. Communication across the border was difficult, although the situation had improved somewhat over the previous decade. In this state of things Eduard Čech decided to organize an event which would bring together mathematicians from the East and the West. It was an enormous effort on his part and, alas, he was not able to see the fruits of it – he died in 1960. However his efforts were not in vain. His students and colleagues managed to finish what he started and in 1961, 147 mathematicians gathered in Prague for a week devoted to topology. This has started a tradition that every five years mathematicians from all over the world, interested in diverse areas of topology, come to meet in Prague.

The Twelfth Symposium on General Topology and its Relations to Modern Analysis and Algebra – Toposym 2016 – is held in Prague on July 25–29., 2016. The symposium is organized by the Institute of Mathematics of the Czech Academy of Sciences and the Faculty of Mathematics and Physics of the Charles University in Prague. The meeting is held in the lecture halls of the Faculty of Architecture, Czech Technical University in Prague. The symposium is attended by about 250 mathematicians from multiple countries. The program of the symposium consists of 28 invited lectures delivered by mathematicians selected by the scientific committee, an invited talk delivered by the 2015 Mary Ellen Rudin Young Researcher Award winner, more than 100 selected contributed talks, and about 20 poster presentations.

David Chodounský and Jonathan Verner

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Leandro Aurichi
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Ondřej Kalenda
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Piotr Koszmider
Mikolaj Krupski
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INVITED TALKS

The program of Toposym 2016 contains 28 invited talks delivered by mathematicians selected by the scientific committee of the symposium. An invited talk is also delivered by the 2015 Mary Ellen Rudin Young Researcher Award winner, Yinhe Peng (University of Toronto, Canada). All invited talks are 50 minutes long, most of them presented in 2 parallel sessions. The invited talks are video-recorded and slides for the talk are available on the conference website.

Topological Groups, Coset Spaces, and their Remainders

Alexander V. Arhangel'skii

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Suppose G is a topological group and H is a closed subgroup of G. Then G/H is the quotient space of G, that is, members of G/H are left cosets xH, where $x \in G$, and the topology is the quotient topology. The space G/H is homogeneous. "A space" stands for "a Tychonoff space". A space X is *metric-friendly* if there exists a σ-compact subspace Y of X such that $X \setminus U$ is a Lindelöf p-space, for every open neighbourhood U of Y in X, and the following two conditions are satisfied:

1. For every countable subset A of X, the closure of A in X is a Lindel p-space.
2. For every subset A of X such that $|A| \leq 2^\omega$, the closure of A in X is a Lindelöf Σ-space.

Theorem *Every remainder of any paracompact p-space is metric-friendly.*

A coset space $X = G/H$ is *compactly-fibered* if H is compact.

Theorem *For every compactly-fibered coset space $X = G/H$, either each remainder of X is metric-friendly, or each remainder of X is pseudocompact.*

Theorem *Suppose X is a compactly-fibered coset space, and $Y = bX \setminus X$ is a remainder of X. Then the following conditions are equivalent: (1) Y is metacompact; (2) Y is paralindelöf; (3) Y is Dieudonné complete; (4) Y is Lindelöf; (5) Y is metric-friendly.*

An internal characterization for productively Lindelöf spaces

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A topological space X is said to be productively Lindelöf if $X \times Y$ is Lindelöf for every Lindelöf space Y. We present an internal characterization for the productively Lindelöf property, thus answering a long-standing problem attributed to Tamano. The technique used is to define a topology over the family of all the open coverings of X.

The Zariski topology of a group

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Markov introduced in 1944 the notion of an *algebraic set* of a group G as an arbitrary intersection of finite unions of solution-sets of appropriately defined one variable equations over G. He called *unconditionally closed* a subset of G that is closed in any Hausdorff group topology on G, observing that algebraic sets are unconditionally closed. He proved that these two notions coincide for countable groups and raised the question on whether this remains true in general. One can define two T_1 topologies $\mathfrak{Z}_G$ (*Zariski topology*) and $\mathfrak{M}_G$ (*Markov topology*) on G having as all closed sets all algebraic, and all unconditionally closed sets, respectively. Then $\mathfrak{Z}_G \leq \mathfrak{M}_G$ and Markov's question is equivalent to asking whether $\mathfrak{Z}_G = \mathfrak{M}_G$. The equality $\mathfrak{Z}_G = \mathfrak{M}_G$ for Abelian G was proved by Perel′man, while a counterexample in the general case is attributed to Hesse, but both results were never published. Groups G with discrete $\mathfrak{M}_G$, answering Markov problem on the existence of infinite non-topologizable group, were built by Shelah (under CH), while infinite groups G with discrete $\mathfrak{Z}_G$ were built (in ZFC) by Ol′shankij and his school.

The aim of the talk is to present recent applications of these topologies:

1. some joint results with D. Shakhmatov concerning other problems of Markov for Abelian groups (among them, the description of the potentially dense subsets, a positive solution of Markov's conjecture on connected topologization, etc.);

2. some recent results of Banakh, Chang, Gartside, Glyn, Guran, Megrelishvili, Polev, Protasov, Sipacheva and Toller in the non-Abelian case.

PFA(S) implies there are many S-names

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PFA(S) is the statement that there is a coherent Souslin tree S and that Martin's Axiom holds for the class of proper S-preserving posets. The weaker statement SA_{ω_1} for S-preserving ccc posets was introduced by Larson and Todorcevic to solve Katetov's problem. Todorcevic introduced PFA(S) and showed its consistency from a supercompact cardinal. Todorcevic, Larson-Tall, and Tall have proven many consequences of PFA(S)[S] – i.e. statements holding in the forcing extension by S. We continue the investigation.

We cite two examples that are joint results with F. Tall.

Theorem *PFA(S) implies that forcing with S produces a model in which perfect sequential preimages of ω_1 contain a copy of ω_1.*

Another example, using the stronger principle MM(S) and using very strong stationary set reflection, is

Theorem *MM(S) implies that after forcing with S locally compact normal spaces are $\aleph_1$-CWH.*

Question *Can the hypotheses be weakened to PFA(S)?*

[1] The author was partially supported by the NSF.

Weak diamonds and topology

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We shall review recent applications of the parametrized diamond principles in topology.

Measuring noncompactness and discontinuity

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There are several ways how to measure (relative) noncompactness of sets and operators in Banach spaces. I will survey and compare measures of noncompactness in several topologies. Further, there is a close connection between various types of compactness in Banach spaces with certain types of continuity. For example, the Arzelà–Ascoli theorem reveals a relation of continuity and norm-compactness and some Grothendieck's theorems show connections between weak compactness and Mackey continuity and between Mackey compactness and weak sequential continuity. I will address a quantitative approach to these relationships, i.e., possible strengthenings of certain implications and equivalences to suitable inequalities. The lecture will be based mainly on the papers [1],[2],[3],[4].

[1] B. Cascales, O. F. K. Kalenda, and J. Spurný, *A quantitative version of James's compactness theorem*, Proc. Edinb. Math. Soc. (2) **55** (2012), no. 2, 369–386

[2] M. Kačena, O. F. K. Kalenda, and J. Spurný, *Quantitative Dunford-Pettis property*, Adv. Math. **234** (2013), 488–527

[3] O. F. K. Kalenda and J. Spurný, *Quantification of the reciprocal Dunford-Pettis property*, Studia Math. **210** (2012), no. 3, 261–278

[4] O. F. K. Kalenda and J. Spurný, *On quantitative Schur and Dunford-Pettis properties*, Bull. Aust. Math. Soc. **91** (2015), no. 3, 471–486

[1] The author was partially supported by the grant GAČR P201/12/0290.

Descriptive Graph Combinatorics

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This talk is about a relatively new subject, developed in the last two decades or so, which is at the interface of descriptive set theory and graph theory but also has interesting connections with other areas such as ergodic theory and probability theory.

The object of study is the theory of definable graphs, usually Borel or analytic, on Polish spaces and one investigates how combinatorial concepts, such as colorings and matchings, behave under definability constraints, i.e., when they are required to be definable or perhaps well-behaved in the topological or measure theoretic sense.

Noncommutative scattered locally compact spaces

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The passage from commutative to noncommutative mathematics has stimulated a big part of mathematical research since the mid 20th century which resulted in the unprecedented impact of this programme on the shape of today's mathematics. Quite central in it were the noncommutative geometry and topology but a noncommutative set-theoretic topology is being developed only recently.

In this talk I we will try to explain some of the challenges of the noncommutative set-theoretic topology focusing on the noncommutative analogues of scattered compact or locally compact Hausdorff spaces known as scattered C^*-algebras. These objects have been investigated since the 80ties but in separation from the classical commutative case (ordinals, Ψ-spaces, ladder system spaces, thin-(very) tall spaces etc).

The talk is based on a joint project with Saeed Ghasemi (IM PAN) in which we developed the noncommutative Cantor–Bendixson derivative on the level of the algebra in the analogy to this kind of derivative for superatomic Boolean algebras and we looked at the basic constructions and problems corresponding to the classical programme of "cardinal sequences" for scattered compact spaces (or equivalently superatomic Boolean algebras), in particular such as thin-tall algebras or Ψ-spaces. The results obtained show increased combinatorial difficulties but also contribute to the theory of nonseparable C^*-algebras providing new examples.

The only noncommutative background which will be needed to follow the talk concerns the multiplication of matrices.

Squares of function spaces and function spaces on squares

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For a Tychonoff space X, by $C_p(X)$ we denote the space of all continuous real-valued functions on X endowed with the topology of pointwise convergence. In the 1980s A.V. Arhangel'skii posed a problem whether for a (metrizable/compact) space X the space $C_p(X)$ is homeomorphic to its own square $C_p(X) \times C_p(X)$. Similar questions can be also formulated for linear homeomorphisms or linear continuous surjections from $C_p(X)$ onto $C_p(X) \times C_p(X)$ or onto $C_p(X \times X)$.

In my talk I will present some recent developments concerning these type of questions. In particular, I will show a metrizable counterexample to the problem of Arhangel'skii (the counterexample was obtained together with W. Marciszewski). It turns out that some counterexamples can naturally come from continuum theory, e.g. if M is a Cook continuum, then $C_p(M)$ cannot be mapped linearly onto $C_p(M) \times C_p(M)$.

I will also discuss some related problems and results (obtained in collaboration with A. Leiderman) concerning embeddings of free (abelian) topological groups of the form $A(X \times X)$ into $A(X)$, where X is a metric continuum.

[1] The author was partially supported by the Polish National Science Center research grant UMO-2012/07/N/ST1/03525

Generic objects in topology

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We shall present a new, category-theoretic, concept of a *generic object* in various subcategories of the category of compact Hausdorff spaces. Roughly speaking, a topological space U is *generic* over a class (category) of topological spaces if one of the players has a winning strategy in a natural infinite game producing an inverse sequence converging to U. For example, consider the unit interval $[0,1]$ and assume that two players alternately choose continuous surjections $[0,1] \leftarrow [0,1]$. After infinitely many steps, an inverse sequence of continuous surjections of the unit interval is produced. One can look at its inverse limit as the result of the play. It turns out that each of the players has a strategy leading to the pseudo-arc, a rather special and well-known chainable continuum. By this way, the pseudo-arc is generic in the category of continuous surjections of the unit interval.

We shall present a general framework capturing objects as above. As a byproduct, we present a new proof of the homogeneity of spaces like the Cantor set and generalized Baire spaces.

Theorem *Given a cardinal κ, every homeomorphism between closed nowhere dense subsets of κ^ω extends to an auto-homeomorphism of κ^ω.*

The Lelek fan and the Poulsen simplex from Fraïssé sequences

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We describe the Lelek fan and the Poulsen simplex in the Fraïssé-theoretic framework in the context of categories enriched over metric spaces, developed by Kubiś, and derive consequences on their universality and homogeneity. Further, using uniqueness of a Fraïssé sequence in a certain category, we show that for every two countable dense subsets of end-points of the Lelek fan there exists a homeomorphism of the Lelek fan mapping one set onto the other. This strengthens a result of Kawamura, Oversteegen, and Tymchatyn.

This is joint work with Wiesław Kubiś.

The Ramsey property for Banach spaces, Choquet simplices, and their noncommutative analogs

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We compute the *universal minimal flows* of the automorphism groups of structures coming from functional analysis and convex: the Gurarij space $\mathbb{G}$, the Poulsen simplex $\mathbb{P}$, and their noncommutative analogs. The Gurarij space is the unique separable approximately ultrahomogeneous Banach space that contains ℓ_n^∞ for every $n \in \mathbb{N}$, while $\mathbb{P}$ is the unique metrizable Choquet simplex with dense extreme boundary. The group $\mathrm{Aut}(\mathbb{G})$ of surjective linear isometries of $\mathbb{G}$ is shown to be *extremely amenable*, by proving the *approximate Ramsey property* of the class of finite-dimensional Banach spaces. Similarly the stabilizer $\mathrm{Aut}_{\mathrm{p}}(\mathbb{P})$ of an extreme point p of $\mathbb{P}$ is proven to be extremely amenable, by establishing the approximate Ramsey property of the class of Choquet simplices with a distinguished point. It is then deduced that the universal minimal flow of $\mathrm{Aut}(\mathbb{P})$ is $\mathbb{P}$ itself. More generally, we prove that for any closed face F of $\mathbb{P}$, the pointwise stabilizer $\mathrm{Aut}_{\mathrm{F}}(\mathbb{P})$ is *extremely amenable*.

We also provide the natural noncommutative analogs of the results above, formulated in the categories of operator spaces and operator systems. In particular we study the *noncommutative Gurarij space* and the *noncommutative Gurarij system*.

This is a joint work with D. Bartošová, M. Lupini and B. Mbombo.

History, structure, results and problems on hyperspaces and symmetric products

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Given a space X there are several ways to construct a new space $K(X)$ from X. Given a continuum X (compact, connected, metric space) we consider the hyperspaces

1. $2^X = \{A \subset X : A \text{ is nonempty and closed }\}$;
2. $C(X) = \{A \in 2^X : A \text{ is connected }\}$;
3. $C_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ components }\}$; and
4. $F_n(X) = \{A \in 2^X : A \text{ has at most } n \text{ points }\}$

Note that $C(X) = C_1(X)$ and $F_1(X)$ is homeomorphic to X.

A continumm X is a cone provided that there exists a space Z such that X is homeomorphic to the cone of Z. Given a hyperspace $K(X) \in \{2^X, C_n(X), F_n(X)\}$ there are several natural problems in the sructure of Hyperspaces. We discuss three in this talk.

Problem *For which continua X is the hyperspace $K(X)$ a cone.*

Problem *Can the space X be recovered when we know the hyperspace $K(X)$.*

Problem *Determine the homogeneity degree of a hyperspace $K(X)$.*

In this talk we establish the history of these problems, survey what has been done in this direction for the past five years and set several problems still unsolved.

Minimal homeomorphisms of a Cantor space: full groups and invariant measures.

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A homeomorphism g of a Cantor space X is said to be *minimal* when all g-orbits are dense. Its *full group* $[g]$, the group of all homeomorphisms of X which map each g-orbit onto itself, is intimately related to the orbit partition induced by g: two minimal homeomorphisms g, h whose full groups are isomorphic (as abstract groups) are *orbit equivalent*, that is, there exists a homeomorphism φ of X such that the g-orbit of any $x \in X$ is mapped by φ onto the h-orbit of $\varphi(x)$. This is analogous to what happens in the measure-preserving context; in that case, full groups admit a useful Polish group topology. We prove that such is not the case in the topological context and that full groups of minimal homeomorphisms are coanalytic non-Borel subsets of $\mathrm{Homeo}(X)$. We then argue that the *closure* of $[g]$ inside $\mathrm{Homeo}(X)$ is an interesting object of study. Denoting by K_g the set of all g-invariant Borel probability measures on X, a combination of results of Glasner–Weiss and Giordano–Putnam–Skau shows that $\overline{[g]} = \{h \in \mathrm{Homeo}(X) : \forall \mu \in K_g \ h_*\mu = \mu\}$ and that $\overline{[g]}$ is again a complete invariant for orbit equivalence. The question arises of characterizing which sets K of probability measures on X can be realized as the set of all invariant measures for some minimal homeomorphism g. I will describe an answer to that question, extending a result of Akin (corresponding to the case when K is a singleton) and running parallel to some unpublished work of Dahl.

[1] Both authors were partially supported by ANR grant Grupoloco.

Some results and problems on Countable Dense Homogeneous spaces

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All spaces under discussion are separable and metrizable. A topological space X is Countable Dense Homogeneous (abbreviated: CDH) provided that for all countable dense subsets D and E of X there is a homeomorphism $f : X \to X$ such that $f(D) = E$. Well-known example of CDH-spaces include the real line, the Cantor set and the Hilbert cube. We discuss some old and some new problems on CDH-ness. An old and very tough problem is whether every connected Polish CDH-space is locally connected. This is known to be true for locally compact spaces. We provide a partial answer to the problem which has the result for locally compact spaces as a corollary. The old problem whether there is a meager CDH-space was solved by Farah, Hrušák, and Martínez Ranero in ([1]). Their example has size $\aleph_1$. A relatively new result by Hernández-Gutiérrez, Hrušák and van Mill ([2]) is that such spaces exist for every cardinality less than or equal to $\mathfrak{b}$. Such spaces are necessarily λ-sets. That are spaces in which every countable subspace is G_δ. We prove, in joint work with Hrušák, that the existence of a nontrivial connected meager CDH-space is independent of ZFC. The main new result here is that under the Continuum Hypothesis, there is a connected dense CDH-subspace of the Hilbert cube which is a λ-set. Whether such a set exists in the plane, remains an intriguing open problem.

[1] I. Farah, M. Hrušák, and C. M. Ranero, *A countable dense homogeneous set of reals of size* $\aleph_1$, Fund. Math **186** (2005), no. 1, 71–77

[2] R. Hernández-Gutiérrez, M. Hrušák, and J. van Mill, *Countable dense homogeneity and* λ*-sets*, Fund. Math **226** (2014), no. 2, 157–172

On the length of Borel hierarchies

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I will be giving a survey talk on everything I know about the length of Borel hierarchies.

Minimal non σ-scattered linear orders

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A linear order is *scattered* if it does not contain a copy of the rational line and *σ-scattered* if it is a countable union of scattered suborders. In 1971, Laver proved that the class of σ-scattered linear orders is *well quasi-ordered*: if L_i $(i < \infty)$ is a sequence of σ-scattered linear orders, then there is an $i < j$ such that L_i embeds into L_j. At the time, Laver speculated whether his result could be extended in ZFC to a broader hereditary class of linear orders (Baumgartner had shown around the same time that PFA implied that Laver's result could be extended to a broader class of linear orders). An equivalent form of this question can be stated as follows: is there a ZFC example of a linear orders which is minimal with respect to being non σ-scattered? We have proved that this is not the case. We have also shown that PFA^+ can be used to give a rough classification of the non σ-scattered linear orders: every non σ-scattered linear order contains either an Aronszajn type, a real type, or a ladder system indexed by a stationary subset of ω_1, equipped with either the lexicographic or reverse lexicographic order.

[1] This research was supported in part by US NSF grant DMS-1262019

On the classification of one dimensional continua that admit expansive homeomorphisms

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A homeomorphism $h : X \longrightarrow X$ is *expansive* if there exists a $c > 0$ such that for any distinct $x, y \in X$, there exists $n \in \mathbb{Z}$ such that $\mathrm{d}(h^n(x), h^n(y)) > c$. This talk will begin with an overview of the results and open questions on expansive homeomorphisms on one-dimensional continua. Then I will show that if $h : X \longrightarrow X$ is an expansive homeomorphism of a finitely cyclic continuum X, then there exists a periodic indecomposable subcontinuum Y and a $k \in \mathbb{Z}$ such that $h^k|_Y$ is positively continuum-wise fully expansive on Y. A map $f : X \longrightarrow X$ is *positively continuum-wise fully expansive* if for every non-degenerate subcontinuum $A \subset X$, $\lim_{n\to\infty} \mathrm{d}_H(f^n(A), X) = 0$ where d_H is Hausdorff distance. Then I will discuss how the previous result relates to classifying continua that admit expansive homeomorphisms.

Group compactifications and Ramsey-type phenomena

Lionel Nguyen Van Thé[1]

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In this talk, I will illustrate how the existence of fixed points in certain group compactifications allows to derive various combinatorial results of Ramsey-type, in the spirit of [1].

[1] A. S. Kechris, V. G. Pestov, and S. Todorcevic, *Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups*, Geom. Funct. Anal. **15** (2005), no. 1, 106–189

1 The author was partially supported by ESF.

The space of invariant geometric laminations of degree d

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Thurston introduced d-invariant geometric laminations in 1985 (i.e., closed collections of non-crossing chords of the unit disk invariant under the degree d covering map $\sigma(z) = z^d$ on the unit circle) as a tool to model complex polynomials of degree d acting on the complex plane. He showed that if $d = 2$ a quotient of the space of 2-invariant laminations can be modelled by a lamination, which he called QML, so that the quotient of the unit disk which identifies two points if they are joined by a chord in QML is a locally connected continuum $\mathcal{M}_2^{Comb}$ which models the Mandelbrot set $\mathcal{M}_2$ (i.e., there exists a monotone map $m : \mathcal{M}_2 \to \mathcal{M}_2^{Comb}$). Douady has shown that m is a homeomorphism if and only if $\mathcal{M}_2$ is locally connected.

Thurston's proofs make heavy use of the fact that $d = 2$ and do not apply if $d > 2$. This talk will focus on partial generalizations of Thurston's result to the case $d > 2$.

Weak network and a weaker covering property for the basis problem

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We introduce weak network and apply it to the basis problem for first countable regular spaces. For example, we prove that continuous image of any subset of the Sorgenfrey line contains a subset that is either metrizable or Sorgenfrey. We also find a weaker covering property (than weak separation for all finite powers) that guarantees an uncountable metrizable or Sorgenfrey subspace.

[1] The first author was partially supported by the Mary Ellen Rudin Award.
[2] The second author was partially supported by NSERC grant 455916.

Equivariant geometry of Banach spaces and topological groups

Christian Rosendal[1]

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We study uniform and coarse embeddings between Banach spaces and topological groups. A particular focus is put on equivariant embeddings, i.e., continuous cocycles associated to continuous affine isometric actions of topological groups on separable Banach spaces with varying geometry.

[1] The author was partially supported by the NSF..

On hyperfiniteness of boundary actions of hyperbolic groups

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An old result of Dougherty, Jackson and Kechris implies that the boundary action of the free group F_2 induces a hyperfinite equivalence relation. During the talk, I will discuss generalizations of this theorem to the class of hyperbolic groups. The examples discussed will include gruops acting properly and cocompactly on CAT(0) cube complexes.

[1] The authors were partially supported by the NSERC

Projective Fraïssé limits and homogeneity for tuples of points of the pseudoarc

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The pseudoarc is the generic compact, connected, metric space. It can be represented as a canonical quotient of the pre-pseudoarc, a certain projective Fraïssé limit. (Fraïssé theory is a method from classical Model Theory of producing canonical limits of certain families of finite structures.) I will present results on adding and characterizing generic tuples of points in the pre-pseudoarc. These results imply an appropriate partial homogeneity for tuples of points in the pre-pseudoarc. The proof uses tools coming from combinatorics and logic. From the partial homogeneity of the pre-pseudoarc, I will deduce the topological homogeneity for tuples of points in the pseudoarc. I will finish with speculations on what the ultimate homogeneity for the pseudoarc may be and present some results obtained in that direction.

[1] The first author was partially supported by NSF grant DMS-1266189

Pinning Down versus Density

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The *pinning down number* $pd(X)$ of a topological space X is the smallest cardinal κ such that for any neighborhood assignment $U : X \to \tau_X$ there is a set $A \in [X]^\kappa$ with $A \cap U(x) \neq \emptyset$ for all $x \in X$. Clearly, $c(X) \leq pd(X) \leq d(X)$.

The following statements are equivalent: (a) $2^\kappa < \kappa^{+\omega}$ for each cardinal κ; (b) $d(X) = pd(X)$ for each Hausdorff space X; (c) $d(X) = pd(X)$ for each 0-dimensional Hausdorff space X; (d) $d(X) = pd(X)$ for each Abelian topological group X; (e) $d(X) = pd(X)$ for each connected, locally connected, homogeneous, regular space X.

Let (f) be the following statement: $d(X) = pd(X)$ for each connected, Tychonoff space X. We proved that (f) is strictly weaker than (a)-(e) above, but the failure of (f) is still consistent.

We show that the following three statements are *equiconsistent*:

1. There is a singular cardinal λ with $pp(\lambda) > \lambda^+$, i.e. Shelah's Strong Hypothesis fails;
2. there is a 0-dimensional Hausdorff space X such that $|X| = \Delta(X)$ is a regular cardinal and $pd(X) < d(X)$;
3. there is a topological space X such that $|X| = \Delta(X)$ is a regular cardinal and $pd(X) < d(X)$.

We discuss cardinal inequalities involving $pd(X)$.

More on the properties of almost connected pro-Lie groups

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Projective limits of finite-dimensional Lie groups, so called *pro-Lie* groups, form a wide class of topological groups with nice properties. This class, defined by K.H. Hofmann and S. Morris some fifteen years ago, contains all Lie groups, is closed under arbitrary products and closed subgroups. All compact groups, locally compact Abelian groups, and connected locally compact groups are pro-Lie groups.

Following Hofmann and Morris, we say that a topological group G is *almost connected* if the quotient group of G by the connected component of its identity is compact. Almost connected pro-Lie groups constitute a proper subclass of pro-Lie groups. For example, it is easy to verify that every almost connected pro-Lie group is *ω-narrow*, which means that the group can be covered by countably many translates of every neighborhood of its identity.

It turns out that all almost connected pro-Lie groups as well as their continuous homomorphic images are *$\mathbb{R}$-factorizable* and *ω-cellular* (i.e. every family of G_δ-sets contains a countable subfamily whose union is dense in the union of the whole family). We will also present a general result which implies, in particular, that if a topological group G contains a compact invariant subgroup K such that the quotient group G/K is an almost connected pro-Lie group, then G is $\mathbb{R}$-factorizable and ω-cellular.

[1] The second author was partially supported by the CONACyT of Mexico, grant number CB-2012-01 178103.

More on the structure theory of the compact sets of the first Baire class

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The structure theory of compact sets of Barie-class-1 function is an already rich theory with notable applications. This will be an overview of this theory with emphasis to more recent results and open problems.

Lindelöf spaces and large cardinals

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An important problem about Lindelöf spaces is the Arhangel'skii's problem. Arhangel'skii ([1]) asked if there is a Hausdorff Lindelöf space with points G_δ which has cardinality $> 2^{\aleph_0}$. Shelah ([2]) showed the consistency of the existence of such a space, but the consistency of the non-existence is still unknown. Scheepers and Tall ([3]) introduced indestructibly Lindelöf spaces, which can be characterized both by a topological game of length ω_1 and by σ-closed forcing. Under a large cardinal assumption, they showed the consistency of no indestructibly Lindelöf space with points G_δ and of cardinality $> 2^{\aleph_0}$. Dias-Tall proved that the non-existence of such an indestructibly Lindelöf space is a large cardinal property. These results indicate some connection between large cardinals and Arhangel'skii's problem.

In this talk, we will present more connections between large cardinals and Lindelöf spaces with large cardinality but small pseudocharacter. We show that if $\square(\omega_2)$ holds, then there is a Lindelöf space with cardinality ω_2 and points G_δ. We also show that, under $V = L$, for each cardinal κ there is a space with cardinality κ^{++} but Lindelöf degree and pseudocharacter κ.

[1] A. Arhangel'skii, *On the cardinality of bicompacta satisfying the first axiom of countability*, Soviet Math. Dokl **10** (1969), no. 4, 951–955

[2] S. Shelah, On some problems in general topology, in *Set theory (Boise, ID, 1992–1994)* , number 192 in Contemp. Math. (Amer. Math. Soc., Providence, RI), p. 91–101.

[3] M. Scheepers and F. D. Tall, *Lindelöf indestructibility, topological games and selection principles*, Fund. Math. **210** (2010), no. 1, 1–46

[1] The author was supported by JSPS KAKENHI grant No. 15K17587 and 15K17587.

Universal Minimal Recurrence

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One of the central notions in topological dynamics is that of recurrence and many kinds of recurrence have been studied during the last few decades. I will discuss a few of these and some new results obtained in joint work with Eli Glasner. In particular I will present a new concept, that of a universal minimal recurrence set, that was motivated by an old paper of P. Erdös and A. Stone.

CONTRIBUTED TALKS

The program of Toposym 2016 contains more than 100 contributed talks. The accepted talks were selected by the scientific committee of the symposium. All invited talks are 30 minutes long, most of them presented in 5 parallel sessions. Slides for the talks are available on the conference website.

On the class of all spaces whose product with every paracompact space is paracompact

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The general question is to characterize the class P of all spaces whose Cartesian product with every paracompact space is paracompact and to verify whether the class is closed with respect to closed mappings and the product of countably many copies of X is paracompact provided that X belongs to P.

R. Telgarsky, in connection with the productivity of paracompactness, introduced a $G(DC, X)$ game of two players in a topological space X and proved that if X is a paracompact space and the first player has a winning strategy then X belongs to P.H. Tanaka proved that if X satisfies the above mentioned assumptions then also the product of countably many copies of X is paracompact. I noticed that a closed image of such a space X has the same properties.

These results led me to a conjecture which I called the Telgarsky conjecture:

Conjecture *A paracompact space X belongs to P if and only if the first player of the $G(DC, X)$ game has a winning strategy.*

Some results supporting this conjecture have been obtained.

During my talk I will focus on some characterizations of projections of X onto the first α coordinates provided that X belongs to P and can be embedded in the product of ω_1 copies of M, where M is a metrizable space and α is less than ω_1.

[1] The author was partially supported by the ESF.

Shore and Non-Block Points in the Stone–Čech Remainder

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When X is a Hausdorff continuum we call $p \in X$ a *proper non-block point* to mean $\kappa(p) - p$ has a dense continuum-component. We call p a *proper shore point* to mean X is in the closure of the subspace

$$\{K \in C(X) : K \subset \kappa(p) - p\}.$$

Non-block points are always shore points but not vice-versa.

There is a well-known correspondence between the composants of $\mathbb{H}^*$ and the near-coherence classes of ω^*. We use this to characterise the proper non-block points of $\mathbb{H}^*$.

Theorem *Suppose the composant $E \subset \mathbb{H}$ corresponds to the near-coherence class $\mathcal{E} \subset \omega^*$. The following are equivalent: (1) E has a proper non-block point. (2) Every point of E is a proper non-block point. (3) $\mathcal{E}$ has a Q-point.*

It follows under Near Coherence of Filters that $\mathbb{H}^*$ is an indecomposable Hausdorff continuum with exactly one composant and no non-block points. This partially answers a question of the author.

We also prove that, while the presence of non-block points in $\mathbb{H}^*$ is axiom-sensitive, shore points always exist in ZFC.

Theorem *Every point of $\mathbb{H}^*$ is a proper shore point.*

[1] This research was supported by the Irish Research Council Postgraduate Scholarship Scheme grant number GOIPG/2015/2744.

Hyperspaces of Euclidean spaces in the Gromov–Hausdorff metric

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The Gromov–Hausdorff distance d_{GH} is a useful tool for studying topological properties of families of metric spaces. For two compact metric spaces X and Y the number $d_{GH}(X, Y)$ is defined to be the infimum of all Hausdorff distances $d_H(i(X), j(Y))$ for all metric spaces M and all isometric embeddings $i : X \to M$ and $j : Y \to M$.

Clearly, the Gromov–Hausdorff distance between isometric spaces is zero; it is a metric on the family GH of isometry classes of compact metric spaces. The metric space $(\mathrm{GH}, \mathrm{d_{GH}})$ is called the Gromov–Hausdorff hyperspace.

This talk is devoted to the subspace $\mathrm{GH}(\mathbb{R}^n)$ of GH consisting of the classes $[E] \in$GH whose representative E is a metric subspace of the Euclidean space $\mathbb{R}^n$, $n \geq 1$. $\mathrm{GH}(\mathbb{R}^n)$ is called the Gromov–Hausdorff hyperspace of $\mathbb{R}^n$. One of the main results of this talk asserts that $\mathrm{GH}(\mathbb{R}^n)$ is homeomorphic to the orbit space $2^{\mathbb{R}^n}/E(n)$, where $2^{\mathbb{R}^n}$ is the hyperspace of all nonempty compact subsets of $\mathbb{R}^n$ endowed with the Hausdorff metric and $E(n)$ is the isometry group of $\mathbb{R}^n$. This is applied to prove that $\mathrm{GH}(\mathbb{R}^n)$ is homeomorphic to the Hilbert cube with a removed point.

[1] The author was partially supported by the UNAM grant PAPIIT-IN101314

Planar embeddings of unimodal inverse limit spaces

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We study the family $\{X_s\}_{s\in[0,1]}$ of inverse limit spaces with tent map bonding maps $T_s : [0,1] \to [0,1]$, $T_s(x) = \min\{sx, s(1-x)\}$. It is a well known fact that X_s are chainable continua. The study of planar embeddings of chainable continua dates back to 1951 when Bing proved that every chainable continuum can be embedded in the plane. The first explicit class of planar embeddings of X_s was given by Brucks and Diamond in 1995 and Bruin in 1999. Recently, Boyland, de Carvalho and Hall constructed a family of continuously varying family of disk homeomorphisms having X_s as global attracting sets. For certain parameters s, continua X_s have a very rich local structure so it would be interesting to see what kind of planar embeddings of complicated X_s are possible. In this talk we will demonstrate the method of explicit construction of uncountably many non-equivalent planar embeddings of X_s using the description of X_s arising from the symbolic dynamics of T_s. We prove the following

Theorem *For every $s \in [0,1]$ and every point $x \in X_s$ there exists an embedding of X_s in the plane such that x is accessible from the complement.*

On quasi-convex null sequences

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Quasi-convex sets in Abelian topological groups play an important role, comparable with convex sets in topological vector spaces. It is a consequence of the Hahn–Banach theorem that a symmetric, closed, convex set in a real locally convex vector space is a quasi-convex subset of the underlying topological group. So it is possible to find quasi--convex sets of cardinality $\geq \mathfrak{c}$. On the other hand, there exist finite quasi-convex sets.

In this talk we will present known and new results on Abelian topological groups which admit a non–trivial quasi-convex null sequence.

Given an Abelian topological group (G, τ) and a quasi-convex null sequence N in the dual group $(G, \tau)^\wedge$ one can ask whether the topology of uniform convergence on N is compatible with τ. This question is of importance when the Mackey topology on a group, i.e. the finest locally quasi-convex group topology on G giving rise to $(G, \tau)^\wedge$, is studied.

On cohomological properties of remainders

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In the paper are studied cohomological properties of remainders of Stone–Čech compactifications. Let $H^n_{\infty,f}(X,A;G)$ be the n-dimensional border cohomology group of closed pair (X,A) of normal spaces with coefficients in Abelian group G based on the set of all extendable fringes of normal space X [1].

The border cohomological dimension $d^\infty_f(X;G)$ is defined to be the smallest integer n such that, whenever $m \geq n$ and A is a closed subset in X, then the homomorphism $i^* : H^n_{\infty,f}(X;G) \to H^n_{\infty,f}(A;G)$, induced by the inclusion $i : A \to X$, is onto.

Let $H^n_f(X,A;G)$ be the n-dimensional Čech cohomology group of closed pair of normal spaces and let $d_f(X;G)$ be the cohomological dimension of normal space X [2].

Theorem *Let A be a closed subset of metrizable space X. Then*

$$H^n_f(\beta X\backslash X, \beta A\backslash A;G) = H^n_{\infty,f}(X,A;G),$$

$$d^\infty_f(X;G) \leq d_f(\beta X\backslash X;G).$$

Remark Such type result also is true for Čech complete spaces and spaces with bicompact axiom of countability [1].

[1] Y. M. Smirnov, *On the dimension of increments of bicompact extensions of proximity spaces and topological spaces*, Mat. Sb. (N.S.) (1966), 141–160

[2] K. Nagami, *Dimension theory*, Academic Press, New York (1970)

[1] The authors supported by grant FR/233/5-103/14 from Shota Rustaveli National Science Foundation (SRNSF)

$\mathfrak{G}$-Bases in free objects of Topological Algebra

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A topological space X has a *local $\mathfrak{G}$-base* if every point $x \in X$ has a neighborhood base $(U_\alpha)_{\alpha\in\omega^\omega}$ such that $U_\beta \subset U_\alpha$ for all $\alpha \le \beta$ in ω^ω.

Theorem *For a Tychonoff space X the following conditions are equivalent:*

1. *The free Abelian topological group $A(X)$ of X has a local $\mathfrak{G}$-base.*
2. *The free Boolean topological group $B(X)$ of X has a local $\mathfrak{G}$-base.*
3. *The universal uniformity $\mathcal{U}(X)$ of X has a $\mathfrak{G}$-base.*

If X is first-countable and perfectly normal, then (1)–(3) are equivalent to:

4. *X is metrizable and has σ-compact set X' of non-isolated points.*

Theorem *For a Tychonoff space X the following conditions are equivalent:*

1. *The free locally convex space $L(X)$ of X has a local $\mathfrak{G}$-base.*
2. *The free topological vector space $V(X)$ of X has a local $\mathfrak{G}$-base.*
3. *The universal uniformity $\mathcal{U}(X)$ of X has a $\mathfrak{G}$-base and the function space $C(X)$ is ω^ω-dominated (in $\mathbb{R}^X$).*

The conditions (1)-(3) imply (and if X is not a P-space, are equivalent to):
(4) the free topological group $F(X)$ of X has a local $\mathfrak{G}$-base.
If X is first-countable and perfectly normal, then (1)–(3) are equivalent to metrizability and σ-compactness of X.

On a problem of Ellis and Pestov's conjecture

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The problems of Ellis comes purely from topological dynamics, asking whether the greatest ambit of a topological group is isomorphic to the Ellis enveloping semigroup of its universal minimal flow. It was answered in the negative for the discrete group of integers by Glasner ([1]) and for the group of order preserving automorphisms of the rationals by Pestov ([2]) with use of Ramsey theory. Extremely amenable groups, that is, groups with trivial universal minimal flows form a rich class of counterexamples to Ellis' problem. Pestov conjectured that a positive answer holds only for precompact groups, that is, subgroups of compact groups. For precompact groups the greatest ambit and the universal minimal flow coincide. We confirm this conjecture for S_∞, the group of permutations of a countable set, and relate that to Ramsey theory and algebra on ultrafilters.

[1] E. Glasner, *On minimal actions of polish groups*, Topology and its Applications **85** (1998), no. 1, 119–125
[2] V. Pestov, *On free actions, minimal flows, and a problem by Ellis*, The Transactions of the American Mathematical Society **350** (1998), no. 10, 4149–4165

[1] The first author was supported by FAPESP 2013/14458-9

Tree sums and maximal connected I-spaces

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If $\mathcal{P}$ is a property of topological spaces, we say that a topological space $\langle X, \tau \rangle$ (or the corresponding topology τ) is *maximal* $\mathcal{P}$ if it has the property $\mathcal{P}$, but no strictly finer topology $\tau^* > \tau$ has the property $\mathcal{P}$. We are interested in the case where $\mathcal{P}$ means connectedness, i.e. in *maximal connected topologies*.

We show how the property of being maximal connected (and also *essentially connected* and *strongly connected*) is preserved by the construction of so-called *tree sums* of topological spaces under certain conditions.

We also recall the characterization of *finitely generated* maximal connected spaces and reformulate it in the language of *specialization preorder* and graphs, from which it is imminent that finitely generated maximal connected spaces are precisely certain tree sums of copies of the Siepiński space.

Finally, we note that several classes of topological spaces are equivalent to the class of *I-spaces* in the realm of maximal connected spaces, and that maximal connected I-spaces include the class of finitely generated maximal connected spaces. We present several results towards characterization of maximal connected I-spaces.

[1] The author was supported by the grant SVV-2016-260336 of Charles University in Prague and by the University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics (MathMAC).

On hyperstructures in topological categories

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We will propose and discuss a new approach to define hyperstructures, such as Vietoris hyperspaces in **Top**, which works in every cartesian closed topological category, and so applies to every topological category, using it's topological universe hull. The key tool is the embedding μ_X together with the projections π_A in the situation

$$C(X,Y) \xrightarrow{\mu_X} K(Y)^{K(X)} \cong \prod_{A \in K(X)} K(Y)_A \xrightarrow{\pi_A} (K(Y), \sigma_V)$$

as is explained in [1]:

Theorem *Let (Y,σ) be an infinite T_3-space. For every topological space let $C(X,Y)$ be equipped with compact-open topology. Let $\mathcal{B}$ be a class of topological spaces, that contains the Stone–Čech-compactification of a discrete space with cardinality at least card(Y).*

Then the Vietoris topology σ_V on $K(Y)$ is the final topology w.r.t. all $\pi_A \circ \mu_{(X,\tau)}$, $(X,\tau) \in \mathcal{B}$, $A \in K(X,\tau)$.

Using this as role model, we can examine appropriate hyperstructures for pseudotopological spaces, semiuniform convergence spaces or multifilter spaces, for example.

[1] R. Bartsch, *Vietoris hyperspaces as quotients of natural function spaces*, Rostock. Math. Kolloq. (2014/15), no. 69, 55–66

Cohomology of Ordinals

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We round out the picture of the cohomology groups of an arbitrary ordinal δ. This is, at heart, a question of the existence of nontrivial coherent families of functions indexed by δ, and of their higher-order analogues.

[1] The author was partially supported by the NSF.

Strong shape and homology of continuous maps

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In paper [1] the fiber resolution and fiber expansion of continuous maps are defined and it is shown that any fiber resolution is a fiber expansion. In this paper we have defined a strong fiber expansion. We have shown that any fiber resolution is a strong fiber expansion. Besides, we have proved an analogous lemma of the Main Lemma about strong expansion [2]. Using the obtained results and methods of strong shape theory [2] we have constructed a strong fiber shape category of maps of compact metric spaces.

In the second part of this paper, we have constructed the strong homological functor from the strong shape category of maps of compact metric spaces to the category of sequences of Abelian groups and level mophisms. Using the obtained results we have defined the homological functor $\mathbf{H} : \mathbf{Mor_{CM}} \to \mathbf{Ab}$, which is strong shape invariant and has the semi-continuous property. Besides, we give an example of a map $f : X \to Y$ with trivial spectral homology and non-trivial strong homology groups.

[1] V. Baladze, *Fiber shape theory*, Proc. A. Razmadze Math. Inst. (2003), no. 132, 1–70

[2] S. Mardešić, *Strong shape and homology*, Springer-Verlag, Berlin (2000)

[1] The authors supported by grant FR/233/5-103/14 from Shota Rustaveli National Science Foundation (SRNSF)

On the neigborhoods of the diagonal

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A topological space X is called *uniformly normal* if the family of all symmetric neighborhoods of the diagonal $\Delta \subset X \times X$ forms a uniformity on X. The neighborhood of the diagonal is any subset, whose interior contains the diagonal. It is well-known that all uniformly normal spaces are collectionwise normal. H. H. Corson ([1]) showed that every Σ-product of complete separable metric spaces is uniformly normal. A. P. Kombarov ([2]) showed that every Σ-product of Lindelöf Čech-complete spaces of countable tightness is uniformly normal.

We proved the next generalization:

Theorem *Σ-product of Lindelöf p-spaces of countable tightness is uniformly normal.*

[1] H. H. Corson, *Normality in subsets of product spaces*, Amer. J. Math. **81** (1959), 785–796

[2] A. Kombarov, *On the product of normal spaces*, Soviet. Math. Dokl **13** (1972), no. 4, 1068–1071

Near-rings of Continuous Functions and Primeness

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If $(G, +)$ is a topological group, define

$$N(G) := \{a : G \to G : a \text{ is continuous}\}$$

and

$$N_0(G) := \{a \in N_0(G) : a(0) = 0\}.$$

Then $N(G)$ and $N_0(G)$ are a near-ring and a zero-symmetric near-ring, respectively. In the case that the topology on G is discrete, we will write $M(G)$ and $M_0(G)$, instead of $N(G)$ and $N_0(G)$, respectively. The structure of $M(G)$ and $M_0(G)$ hes been extensively investigated for almost as long as near-rings themselves have been studied. The implications of the topology on G for the structures of $N(G)$ and $N_0(G)$ have been investigated since the early 1970's.

Several different definitions of primeness exist in the literature of near-rings, all of which generalise the classical concept for associative rings. All of these definitions give rise to prime radicals in the varieties of zero-symmetric and all near-rings.

In this presentation we will investigate the effect of the topology on G on the primeness of $N_0(G)$, and the associated radicals. In particular, we will consider the effects of connectedness, arcwise connectedness and 0-dimensionality. It turns out that the topology has a profound effect, and the results can be very different for the same G with different topologies.

[1] The author was supported by the South African National Research Foundation

Compactification of uniformly continuous mappings

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At the beginning a notion of compactification for a continuous mappings had been introduced and studied in [1], [2], [3], [4]. The notions of uniformly perfect and complete mappings had been introduced and researched in earlier works of author, collected and improved in [5]. Now the well-known Henriksen and Isbell theorem [6] and some results, concerning of points of closedness and perfectness for continuous mappings [7], their analogues for uniformly continuous mappings have been obtained by the author.

[1] G. L. Cain, *Compactifications of mappings*, Proc. Amer. Math. Soc. **23** (1969), no. 2, 298–303

[2] G. T. Whyburn, *A unified space of mappings*, Trans. Amer. Soc. **74** (1953), 344–350

[3] B. A. Pasynkov, *On extending onto mappings some concepts and statements concerning spaces*, Mappings and functors (1984), 72–102 in Russian.

[4] V. M. Ulyanov, *On compactifications of countable character and absolutes*, Matem. Sb. **98** (1975), no. 2, 223–254 in Russian.

[5] A. A. Borubaev, *Uniform topology*. (Ilim, 2013). in Russian.

[6] M. Henriksen and J. R. Isbell, *Some properties of compactifications*, Duke Math. J. **25** (1958), 83–106

[7] R. N. Ormotsadze, *On perfect maps*, Comment. Academy of Sciences of the GSSR **119** (1985), no. 1, 25–28 in Russian.

Quotients of the shift map (for frogs)

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The shift map σ on $\omega^* = \beta\omega - \omega$ is the continuous self-map of ω^* induced by the function $n \to n+1$ on ω. Given a compact Hausdorff space X and a continuous function $f : X \to X$, we say that (X,f) is a quotient of (ω^*,σ) whenever there is a continuous surjection $Q : \omega^* \to X$ such that $Q \circ \sigma = \sigma \circ f$.

For spaces of weight at most $\aleph_1$, (X,f) is a quotient of (ω^*,σ) if and only if f is weakly incompressible (which means that no nontrivial open $U \subseteq X$ has $f(\overline{U}) \subseteq U$). Under CH, this gives a complete characterization of the quotients of (ω^*,σ) and implies, for example, that (ω^*,σ^{-1}) is a quotient of (ω^*,σ).

In this talk, I will outline and discuss the proof of this theorem. The main tools used are transfinite recursion, elementary submodels, and a detailed look at the quotients of (ω^*,σ) and $(\beta\omega,\sigma)$.

Arhangelskii's alpha properties of $C_p(X)$ and covering properties of X

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I will present characterizations of a topological space X, for which the topological space $C_p(X)$ possesses Arhangelskii property (α_1) and (α_2), respectively. The results follow from results obtained by the author and others (the references will be given in my lecture).

Theorem *For a perfectly normal topological space X the following are equivalent:*

1) the set of all real upper semicontinuous functions on X possesses the (α_2) property,

2) X is a $S_1(\Gamma, \Gamma)$-space,

3) X is a wQN-space.*

Theorem *For a perfectly normal topological space X the following are equivalent:*

1) $C_p(X)$ possesses the (α_1) property,

2) the set of real lower semicontinuous functions on X possesses the (α_2) property,

3) the set of γ-covers of X possesses the covering (α_1) property,

4) X is a QN-space,

5) every Borel image of X into Baire space ${}^\omega\omega$ is bounded.

I will present similar result for $C_p(X)$ with the (α_2) property.

[1] The author was supported by the grant 1/0097/16 of VEGA.

Universality of embeddability between groups

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Working in the framework of Borel reducibility we study the quasi-order of embeddability between groups. We prove that the embeddability between countable groups and the topological embeddability between Polish groups are invariantly universal for analytic quasi-orders. The first strengthens a result by Williams [1], while the second improves a result by Ferenczi–Louveau–Rosendal [2]. Most of the techniques we use come from [3] and [1].

[1] J. Williams, *Universal countable Borel quasi-orders*, J. Symb. Log. **79** (2014), no. 3, 928–954.

[2] V. Ferenczi, A. Louveau, and C. Rosendal, *The complexity of classifying separable Banach spaces up to isomorphism*, J. Lond. Math. Soc. (2) **79** (2009), no. 2, 323–345.

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A cardinality bound for T_2 spaces

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For a space X we introduce the cardinal invariant $aL'(X)$, a weakening of the Lindelöf degree $L(X)$, and establish that $|X| \leq 2^{aL'(X)\chi(X)}$ for any T_2 space X. It can be shown that a) $aL'(X) = \aleph_0$ for an H-closed space X, and b) $aL(X) \leq aL'(X) \leq aL_c(X) \leq L(X)$ for a general T_2 space X. Thus, this result gives a common proof that $|X| \leq 2^{\chi(X)}$ for both the class of Lindelöf spaces (Arhangel'skii) and the class of H-closed spaces ([1]). Recall that a space X is H-closed if every open cover of X has a finite subfamily with dense union.

A set $A \subseteq X$ is said to be E-closed if its absolute, EA, is equal to A. Then $aL'(X)$ is defined as the least cardinal κ such that for every cover $\mathcal{U}$ of an E-closed set A by sets open in X there exists $\mathcal{V} \in [\mathcal{U}]^{\leq\kappa}$ such that $A \subset \bigcup_{V\in\mathcal{V}} \overline{V}$.

To put the established bound in context by recall that

1. $|X| \leq 2^{aL_c(X)t(X)\psi_c(X)}$ for a T_2 space X ([2]);
2. $2^{aL(X)\chi(X)}$ is not a bound for the cardinality of all T_2 spaces ([3]);
3. $aL_c(X)$ is not necessarily countable for H-closed X, as witnessed by the Katětov extension $\kappa\omega$ of the countable discrete space ω.

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Topological and algebraic entropy for locally linearly compact vector spaces

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Nowadays various versions of topological entropy are known and studied. Among them we are interested in Hood's extension of the topological entropy introduced by Bowen in 1971 for uniformly continuous self-maps of a metric space. Indeed Hood's definition of topological entropy applies to any continuous endomorphism of a totally disconnected locally compact group.

On the other hand, in 1981 Peters developed an algebraic entropy in an effort to dualize the notion proposed by Bowen and this algebraic entropy was recently extended by Virili to continuous endomorphisms of locally compact Abelian groups.

The class of totally disconnected locally compact Abelian groups contains any locally linearly compact vector space over a finite field. Thus Hood's and Virili's extensions apply to these objects.

The aim of this talk is to introduce both a topological and an algebraic entropy of a continuous endomorphism of a locally linearly compact vector space over an arbitrary discrete field and to discuss some of their properties including the so-called Addition theorem. Finally, it will be shown that these notions of entropy are strictly connected by means of Lefschetz duality in the category of locally linearly compact vector spaces.

The hyperspace of large order arcs

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A continuum is a compact connected metric space. Given a continuum X, let $C(X)$ denote the hyperspace of non-empty subcontinua of X, metrized with the Hausdorff metric. A large order arc in $C(X)$ is an arc $\mathcal{L}$ in $C(X)$ joining X to an element of the form $\{x\}$, for some $x \in X$, and such that for every pair $A, B \in \mathcal{L}$, we have that $A \subset B$ or $B \subset A$. Let $LOA(X)$ denote the set consisting of all large order arcs in $C(X)$, considered as a subspace of $C(C(X))$. In this talk we present several topological properties of $LOA(X)$ and its relations to topological properties of X.

Ultrafilter-completness on zero-sets of uniformly continuous functions.

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Let *ZUnif* be a category whose objects are uniform spaces, and morphisms are *coz*-mappings [1], [2]. Let $Comp(\mathbb{R}\text{-}Comp)$ be the class of compact (realcompact) spaces. Then

$$Comp \equiv \mathfrak{L}([0,1])(\mathbb{R}\text{-}Comp \equiv \mathfrak{L}(u_{\mathbb{R}}\mathbb{R})),$$

where $\mathfrak{L}([0,1])(\mathfrak{L}(u_{\mathbb{R}}\mathbb{R}))$ is the epi-reflective hull of $[0,1](u_{\mathbb{R}}\mathbb{R})$ in the category *ZUnif* [3], [4]. The functors of epi-reflections are the β-like compactification $\beta_u : uX \to \beta_u X$ and realcompactification $v_u : uX \to v_u X$ [5]. Furthermore z_u-*complete* ($\mathbb{R}$-z_u-complete) uniform spaces have been determined, which are in precisely an elements of $\mathfrak{L}([0,1])$ ($\mathfrak{L}(u_{\mathbb{R}}\mathbb{R})$).

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Extendability of the shift homeomorphism of some planar embeddings of unimodal inverse limit spaces

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Let $\varprojlim([0,1],T)$ denote the inverse limit space on the unit interval $[0,1]$ with the unimodal bonding map T. In [1] it was proven that there exists uncountably many different planar embeddings of unimodal inverse limit spaces for a bonding map T with a positive topological entropy.

In order to make $\varprojlim([0,1],T)$ an attractor of some planar diffeomorphism and thus interesting from the dynamical system perspective it should hold that the shift (natural) homeomorphism on $\varprojlim([0,1],T)$ can be extended to the plane.

In this talk I will discuss about the aspect of extendability of the shift homeomorphism of some planar embeddings of $\varprojlim([0,1],T)$.

[1] A. Anušić and J. C. Henk Bruin, *Uncountably many planar embeddings of unimodal inverse limit spaces*, arXiv:1603.03887 (2016)

Homology of Generalized Generalized Configuration Spaces

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We describe an assignment, to each finite simplicial complex S and each topological space X, of a topological space $M(S, X)$. Our construction generalizes the generalized or graph configuration space $M_G(X)$. In particular $M(Cl(G), X) = M_{G^c}(X)$, where G^c stands for the edge-complement of G and $Cl(G)$ is the clique complex of G.

Eastwood–Huggett [1] showed that in case X is a manifold, the Euler characteristic $\chi(M_G(X))$ obeys a deletion-contraction formula, hence $\chi(M_G(\mathbb{CP}^{\lambda-1}))$ is the chromatic polynomial of G evaluated at λ. The characteristic $\chi(M(S, X))$ obeys several similar deletion-contraction formulæ, leading to:

Theorem *If X is a manifold, $\chi(M(S, X))$ is a monic polynomial in $\chi(X)$, whose degree is the number of vertices of S.*

We may thus regard $\chi(M(S, X))$ as a kind of "X-chromatic polynomial" for the simplicial complex S. As in [2], $H_*(M(S, X))$ is an algebraic categorification of this polynomial, and $M(S, X)$ is a topological realization of that categorifcation.

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[1] Partially supported by a Simons Collaboration Grant.

Separable determination in Asplund spaces

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One of the important methods in nonseparable Banach space theory is the "separable reduction". By a separable reduction we usually mean the possibility to extend the validity of a statement from separable spaces to the nonseparable setting without knowing the proof of the statement in the separable case. Experience shows that an optimal method of separable reduction is to construct certain separable subspace of a given non-separable Banach space.

We will show how to handle such a construction using set-theoretical concept of suitable models. We will talk about the method in general and try to explain why this method is very efficient if the space in question is Asplund (i.e. separable subspaces have separable dual). Further, we will show that in Asplund spaces this construction gives not only one subspace, but a "rich family" of them.

All of the above will be presented on a concrete application - namely, we will try to show why (and in what sense) "generalized lushness" is separably determined in Asplund spaces.

Ideal quasi-normal convergence and related notions

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Recently the presenter and D. Chandra began to study the notion of an ideal quasi-normal convergence and some topological notions defined by this convergence. We show that several properties of so introduced notions depend on the ideal and sometimes, they are also equivalent to some important property of the ideal. We present following results.

Theorem *The following are equivalent.*

1. *cof* $(\mathcal{I}) = \kappa$.
2. *For any set* X *and for any sequence of real functions* $f_n \rightarrow^{IQN} f$ *on* X, *there exit sets* X_ξ, $\xi < \kappa$ *such that* $X = \bigcup_{\xi<\kappa} X_\xi$ *and* $f_n \rightarrow^{\mathcal{I}-u} f$ *on each* X_ξ. *Moreover, if* X *is a topological space and* f_n, $n \in \omega$ *are continuous, then the sets* X_ξ *can be chosen to be closed.*

Theorem *The following are equivalent.*

1. *The set* C *is a pseudounion of the ideal* $\mathcal{I}$.
2. *For any set* X *and for any sequence of real functions* $f_n \rightarrow^{IQN} f$ *on* X, *with the control* $(\epsilon_n)_{n\in\omega}$ *there exist sets* $X_k, k \in \omega$ *such that* $X = \bigcup_k X_k$ *and* $f_n \rightarrow^{\langle C\rangle^* - \mathcal{I}} f$ *with same control on each* X_k.

A generalization of the Stone Duality Theorem

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We prove a new duality theorem for the category of precontact algebras which implies the Stone Duality Theorem, its connected version obtained recently in [1], the recent duality theorems from [2] and [3], and some new duality theorems for the category of contact algebras and for the category of complete contact algebras.

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[3] R. Goldblatt and M. Grice, *Mereocompactness and duality for mereotopological spaces*, In: Katalin Bimbo (Ed.), J. Michael Dunn on Information Based Logics (2016 (to appear)), 1–18

[1] The first and the second authors were partially supported by the contract no. 14/2016 "Precontact algebras, dualities and hyperspaces" of the Sofia University "St. Kl. Ohridski" Science Fund.

Topological Ramsey spaces in creature forcing

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Pigeonhole principles play a key role in many aspects of creature forcing. Rosłanowski and Shelah proved a general scheme of Ramsey-type results for partitions of countable sets of finite functions, utilizing and extending Glazer's method of proof of Hindman's Theorem to collections of pure candidates for a large class of creature forcings, proving the existence of idempotent ultrafilters with respect to the related operation. As a consequence, they obtained ultrafilters on countable base set of finite functions which are generated by pure candidates, analogously to ultrafilters generated by infinite block sequences using Hindman's Theorem.

For three specific examples given by Roslanowski and Shelah, we construct topological Ramsey spaces which are dense, hence forcing equivalent, in the collections of pure candidates. This means these spaces satisfy Ellentuck's Theorem in the related exponential topology: Every subset with the property of Baire has the Ramsey property. As a consequence, we recover Rosłanowski and Shelah's result on partition relations for colorings on countable sets of finite functions for these examples. In order to prove that the pigeonhole principle (Axiom A.4) holds for two of the examples, we prove an augmented version of the Product Tree Theorem of Di Prisco, Llopis and Todorcevic.

[1] The first author was partially supported by the NSF.

Completeness of products

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The *completeness number* $*compl\,(\xi)$ of a convergence ξ is the least cardinal κ such that there exists a collection $\mathbb{P}$ of cardinality κ of convergence covers such that each $\mathbb{P}$-fundamental filter is ξ-adherent. A convergence is compact whenever $*compl\,(\xi) = 0$, *locally compactoid* if $*compl\,(\xi) < \aleph_0$ (equivalently, $*compl\,(\xi) = 1$), *Čech-complete* if ξ (is functionally regular) and $*compl\,(\xi) \leq \aleph_0$ (that is, ξ is *countably complete*). Extending a *Frolík theorem* (1960) to convergences, we show that if $*compl\,(\prod \Xi) < \aleph_0$, then $\prod \Xi, = \min(1, *card\,\{\xi : *card\,(\xi) > 0\})$, and, otherwise,

$$*compl\,(\textstyle\prod \Xi) = \sum_{\xi \in \Xi} *compl\,(\xi)\,.$$

In particular, $\prod \Xi$ is compact if and only if ξ is compact for each $\xi \in \Xi$ (the *Tikhonov theorem* for convergence spaces); $\prod \Xi$ is *locally compactoid* if and only if all but finitely many elements of Ξ are compact and the others are locally compactoid; $\prod \Xi$ is *countably complete* if and only if all but countably many elements of Ξ are compact and the others are countably complete.

If $\mathbb{H}$ is a class of filters then the $\mathbb{H}$-*conditional completeness number* $*\,compl_{\mathbb{H}}\,(\xi)$ is the least cardinal κ such that there exists a collection $\mathbb{P}$ of cardinality κ of convergence covers such that each $\mathbb{P}$-fundamental filter from $\mathbb{H}$ is ξ-adherent. In contrast with unconditional completeness. For example, if $\mathbb{H} = \mathbb{F}_1$, then we get a generalization of the *Oxtoby theorem* (1960) that each product of $\mathbb{F}_1$-conditionally countably complete convergences is $\mathbb{F}_1$-conditionally countably complete.

Generic norms and metrics on countable Abelian groups

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For a countable Abelian group G we investigate the Polish space of all norms, resp. invariant metrics, on G. We are mainly interested in generic properties of the space, i.e. properties that are satisfied by all the norms/metrics on G except those coming from a set of first category. We prove that for every countable Abelian group G that is unbounded, i.e. it has elements of arbitrarily high order, there is a dense set of norms on G with which it is isometric to the rational Urysohn space, and a comeager set of norms such that the completion is isometric to the Urysohn space. That generalizes results of Cameron, Vershik, Niemiec and others.

Then we prove that for every G such that $G \cong \bigoplus_{\mathbb{N}} G$ there is a comeager set of norms on G such that all of them give rise to the same metric group after completion. If moreover G is unbounded, then using a result of Melleray and Tsankov we get that the completion is extremely amenable. Among our corollaries is the result that all the known universal Abelian Polish groups, e.g. the Shkarin's group, are extremely amenable.

Cotorsion-free groups from a topological viewpoint

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We present a characterization of cotorsion-free Abelian groups in terms of homomorphisms from fundamental groups of Peano continua. For an open cover $\mathcal{U} \in Cov(X)$ of a space X, we consider the subgroup $\pi(\mathcal{U}, x)$ of $\pi_1(X, x)$, generated by all elements $[\alpha \cdot \beta \cdot \alpha^{-}]$ with $\beta \subseteq U \in \mathcal{U}$. We call a group G *homomorphically Hausdorff relative to* X if for every homomorphism $h : \pi_1(X, x) \to G$,

$$\bigcap_{\mathcal{U} \in Cov(X)} h(\pi(\mathcal{U}, x)) = 1.$$

We call G *Spanier-trivial relative to* X, provided

$$h(\bigcap_{\mathcal{U} \in Cov(X)} \pi(\mathcal{U}, x)) = 1.$$

Theorem *For an Abelian group G, the following are equivalent:*

1. *G is cotorsion-free.*
2. *G is homom. Hausdorff relative to every Peano continuum.*
3. *G is homom. Hausdorff relative to the Hawaiian Earring.*
4. *G is Spanier-trivial relative to the Griffiths twin cone.*

We also calculate the first homology group of the Griffiths twin cone.

[1] The first author was partially supported by the Grant-in-Aid for Scientific Research (C) of Japan (No. 20540097 and 23540110)

[2] The presenting author was partially supported by a grant from the Simons Foundation (No. 245042)

Cardinality of the Ellis semigroup on compact countable metrizable spaces

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In a dynamical system (X, f) the space X will be compact metric, $E(X, f)$ will denote its Ellis semigroup and

$$E(X, f)^* = E(X, f) \setminus \{f^n : n \in \mathbb{N}\}.$$

We analyze the cardinality of $E(X, f)$ for certain compact spaces. Necessary and sufficient conditions for (X, f) are given in order that either $E(X, f)$ or $E(X, f)^*$ be finite. We show that if the set of all periods of the periodic points of (X, f) is infinite, then $E(X, f)$ has at least size $2^{\aleph_0}$. We also prove that if (X, f) has a point with dense orbit and all elements of $E(X, f)^*$ are continuous, then $|E(X, f)^*| \leq |X|$. With respect to a dynamical system of the form $(\omega^2 + 1, f)$, we show if it has a point with dense orbit, then $E(\omega^2 + 1, f)^*$ is countable and all its elements are continuous functions. We give several examples related to the main results.

$\mathcal{I}$-convergence classes

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Let X be a non-empty set. In this talk we consider the class $\mathcal{C}$ consisting of triads $(s, x, \mathcal{I})$, where $s = (s_d)_{d \in D}$ is a net in X, $x \in X$ and $\mathcal{I}$ is an ideal of D. We shall find several properties of $\mathcal{C}$ such that there exists a topology τ for X satisfying the following equivalence: $((s_d)_{d \in D}, x, \mathcal{I}) \in \mathcal{C}$, where $\mathcal{I}$ is a proper D-admissible ideal on D, if and only if $(s_d)_{d \in D}$ $\mathcal{I}$-converges to x relative to the topology τ.

Lifting homeomorphisms from separable quotients of ω^*

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In [1], the authors showed that every isomorphism of between two countable subalgebras of $\mathcal{P}(\omega)/\mathit{fin}$ extends to an automorphism of $\mathcal{P}(\omega)/\mathit{fin}$. Using Stone duality, this result can be viewed as a statement about homeomorphisms between zero-dimensional quotients of ω^*. We generalize this to higher dimensional quotients.

Theorem *Let X and Y be compact metric spaces and let $f : \omega^* \to X$ and $g : \omega^* \to Y$ be continuous and onto. If $\varphi : X \to Y$ is a homeomorphism, then there is a homeomorphism $\overline{\varphi} : \omega^* \to \omega^*$ such that $\varphi \circ f = g \circ \overline{\varphi}$.*

The proof of this theorem uses the fact that every continuous map from ω^* to a metric space extends to a continuous map from $\beta\omega$ to the same metric space.

[1] A. Bella *et al.*, *Embeddings into $\mathcal{P}(\mathbb{N})/\mathit{fin}$ and extension of automorphisms*, Fund. Math. **174** (2002), no. 3, 271–284

[1] The author was partially supported by a grant of the Bonn International Graduate School

Congruence-free compact semigroups

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A classical result of semigroup theory says that a finite *congruence-free* semigroup S (i.e., S has exactly two congruences) without zero such that $\mathsf{card}(S) > 2$ is a simple group. On the other hand, by a *topological* congruence ρ on a topological semigroup S we shall mean an algebraic congruence such that the quotient space S/ρ is a topological semigroup with respect to the quotient topology (notice that an algebraic congruence on a compact semigroup is topological if and only if it is closed). Further, a topological semigroup S is *congruence-free* if the set of its topological congruences is equal to $\{1_S, S\times S\}$. Recall that a topological semigroup S is called *metric* if there exists a subinvariant metric m on S (that is, $m(ca, cb) \leq m(a, b)$ and $m(ac, bc) \leq m(a, b)$ for all $a, b, c \in S$) which determines the topology of S. In this talk, I will present a sketch of the proof of the following result:

Theorem *Every infinite congruence-free compact semigroup S is a connected metric Lie group (so all left and right translations of S are isometries) with cardinality $\mathfrak{c}$.*

In group theory, a *simple* Lie group is a connected locally compact non-Abelian Lie group G which does not have nontrivial *connected* normal subgroups. Clearly, the well-known classification of simple Lie groups has nothing to do with the classification of finite simple groups. On the other hand, it is easy to see that a compact group is congruence-free if and only if it does not have nontrivial *closed* normal subgroups.

Problem *Classify all congruence-free compact groups.*

(see the above theorem)

Topological entropy on totally disconnected locally compact groups

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We study the topological entropy h_{top} for continuous endomorphisms $\phi : G \to G$ of totally disconnected locally compact groups G. In this setting, we prove the Addition Theorem for h_{top} under suitable assumptions, that is, if H is a closed ϕ-invariant subgroup of G and H is either compact or normal in G, then

$$h_{top}(\phi) = h_{top}(\phi \restriction_H) + h_{top}(\bar{\phi}),$$

whenever $\phi \restriction_H$ is surjective and the endomorphism $\bar{\phi} : G/H \to G/H$ induced by ϕ is injective.

As an application we give a dynamical interpretation of the scale $s(\phi)$, by showing that $\log s(\phi)$ is the topological entropy of a suitable map induced by ϕ. We find also necessary and sufficient conditions for the equality $\log s(\phi) = h_{top}(\phi)$ to hold.

Connectedness and generalised inverse limits

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It has recently been shown that a generalised inverse limit over intervals (an inverse limit with upper semicontinuous set-valued functions), is connected if and only if the system does not admit a CC-sequence. We define a related notion, a component base, and show that a GIL is connected if and only if the system does not admit a component base. We demonstrate how this new tool can be applied, and we present a number of new results on connectedness. In particular, results relating to the size and number of components.

g-second countable spaces and the Axiom of Choice

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A topological space X is g-second countable if it has a weak local base $(\mathcal{N}_x)_{x \in X}$ such that $\bigcup_x \mathcal{N}_x$ is countable. Clearly a second countable space is g-second countable, but the reverse is also true for metric spaces.

Using the Axiom of Countable Choice, one can prove that a metric space is g-second countable iff it is separable iff it is Lindelöf. In this talk we will discuss the set-theoretic status of these equivalences as well as other results related with g-second countable and g-first countable spaces.

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Compact spaces with a $\mathbb{P}$-diagonal

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A space has a $\mathbb{P}$-diagonal if the complement of its diagonal in its square has a cover $\{K_f : f \in \mathbb{P}\}$ by compact sets with the property that $f \leq g$ implies $K_f \subseteq K_g$. (The letter $\mathbb{P}$ denotes the set of functions from ω to ω.)

We prove that every compact space with a $\mathbb{P}$-diagonal is metrizable and along the way we prove a Baire category type result for the Cantor cube 2^{ω_1} and covers as above.

Interpolation sets in spaces of continuous functions

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According to S. Hartman and C. Ryll-Nardzewski, a subset S of a topological group G is an I_0 (or *interpolation*) set when every complex-valued bounded function defined on S can be interpolated by an almost periodic function of G. Understanding the properties of interpolation sets is a major topic in the theory of locally compact Abelian (LCA) groups, where there still are many unsolved questions, even for discrete Abelian groups. In this talk, we are concerned with the following variation of this notion: Let X be a topological space and let E be a Banach space. A subset Y of X is called *E-interpolation* (or I_E) set when for every function $g \in E^Y$ with relatively compact range in E, there exists $f \in C(X, E)$ such that $f_{|Y} = g$. We will report on some recent findings about the properties and existence of interpolation sets in spaces of continuous functions and will also discuss on some applications of our results in different settings.

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Continuous Neighborhoods in a Product

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A continuum is a compact connected metric space. Given continua X and Y, we say that the product $X \times Y$ has the full projection implies connected neighborhoods property (fuprocone), provided that for each subcontinuum M of $X \times Y$ with the property that both projections of M on the first and second coordinate are onto, we have that M has arbitrarily small connected neighborhoods in $X \times Y$, that is, for each open subset U of $X \times Y$ such that M is contained in U, there exists an open connected subset V of $X \times Y$ such that M is contained in V and V is contained in U.

Of course, if X and Y are locally connected, then $X \times Y$ has the fuprocone property. In this talk we will mention many examples of nonlocally connected continua X and Y for which $X \times Y$ has de fuprocone property.

We also will talk about some related notions and we will pose some open problems.

Fixed point theorems for maps with various local contraction properties

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Let $\langle X, d\rangle$ be a metric space. A function $f : X \to X$ is *locally contractive* (resp. *locally shrinking*) if for every $x \in X$ there exists $\epsilon_x > 0$ and $\lambda_x \in [0, 1)$ such that $d(f(y_1), f(y_2)) \leq \lambda_x d(y_1, y_2)$ (resp. $d(f(y_1), f(y_2)) < d(y_1, y_2)$) for all distinct $y_1, y_2 \in B(x, \epsilon_x)$. Functions with similar properties are known to have fixed or periodic points for spaces X with certain topological properties (e.g., compactness, connectedness and other). We discuss classic and recently proved fixed/periodic point theorems for several different classes of locally contractive / shrinking functions defined on a variety of metric spaces.

On productively paracompact spaces

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We give necessary conditions and sufficient conditions for productive paracompactness of spaces which are paracompact at infinity. For example, we show that a Tychonoff space with a point-countable base is productively paracompact if, and only if, the space is paracompact and it has a neighbornet (i.e., an assignment of neighborhoods to points) U such that, for every compact set K, the set $U^{-1}K$ is compact.

Baire one functions depending on finitely many coordinates

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Let $(X_n)_{n=1}^{\infty}$ be a sequence of topological spaces, $P = \prod_{n=1}^{\infty} X_n$ and $a = (a_n)_{n\in\mathbb{N}} \in P$ be a point. For every $n \in \mathbb{N}$ and $x \in P$ we put $p_n(x) = (x_1, ..., x_n, a_{n+1}, a_{n+2}, ...)$. We say that a set $A \subseteq P$ *depends on finitely many coordinates*, if there exists $n \in \mathbb{N}$ such that for all $x \in A$ and $y \in P$ the equality $p_n(x) = p_n(y)$ implies $y \in A$. A map $f : X \to Y$ defined on a subspace $X \subseteq P$ is *finitely determined* if it *depends on finitely many coordinates*, i.e., $f(x) = f(y)$ for all $x, y \in X$ with $p_n(x) = p_n(y)$. We denote by $\mathrm{CF}(X, Y)$ the set of all continuous finitely determined maps between X and Y; we write $\mathrm{CF}(X)$ for $\mathrm{CF}(X, \mathbb{R})$.

We answer two questions from [1] and prove, in particular, that every Baire one function on a subspace of a countable perfectly normal product is the pointwise limit of a sequence of continuous functions, each depending on finitely many coordinates. It is proved also that a lower semicontinuous function on a subspace of a countable perfectly normal product is the pointwise limit of an increasing sequence of continuous functions, each depending on finitely many coordinates, if and only if the function has a minorant which depends on finitely many coordinates.

[1] V. Bykov, *On Baire class one functions on a product space*, Topology and its Applications **199** (2016), no. 1-3, 55–62

On dimension of inverse limits with set-valued functions

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Recently, several topological properties of inverse limits of compacta with upper semi-continuous set-valued functions have been studied by many authors. The study of such inverse limits has developed into one of rich topics of geometric topology. There are many differences between the theory of inverse limits with mappings and the theory with set-valued functions. In this paper, we investigate the dimension of inverse limits with set-valued functions. To evaluate the dimension of the inverse limit $\varprojlim \{X_i, f_{i,i+1}\}$ of given inverse sequence $\{X_i, f_{i,i+1}\}_{i=1}^{\infty}$ of compacta with set-valued functions satisfying

$$\dim\{x \in X_{i+1} \mid \dim f_{i,i+1}(x) \geq 1\} \leq 0 \ (i \in \mathbb{N}),$$

we define expand-contract sequences in $\{X_i, f_{i,i+1}\}_{i=1}^{\infty}$ and an index $\tilde{J}(\{X_i, f_{i,i+1}\})$. By use of the index, we prove that

$$\dim \varprojlim \{X_i, f_{i,i+1}\} \leq \tilde{J}(\{X_i, f_{i,i+1}\}) + \sup\{\dim X_i \mid i \in \mathbb{N}\}.$$

Moreover, we evaluate lower bounds of dimensions of some inverse limits of 1-dimensional compacta with set-valued functions.

Random elements of large groups – Continuous case

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We investigate the behavior of random elements of some non-locally--compact groups. Since such groups do not carry a natural measure, we use the following notion introduced by Christensen.

Definition. Christensen Let G be a Polish group. A subset $H \subset G$ is called *Haar null* if there exists a Borel probability measure μ on G and a Borel set $B \supset H$ such that for every $g, h \in G$, $\mu(gBh) = 0$.

It is well-known that the family of Haar null sets form a σ-ideal and that they coincide with the family of measure zero sets (with respect to a left (or equivalently, a right) Haar measure) if G is locally compact.

We examine the group of increasing homeomorphisms of the unit interval and the unit circle. We describe the behavior of the random element of these groups by characterizing the Haar positive (that is, not Haar null) conjugacy classes of these groups. We obtain as a corollary of this characterization that the group of increasing homeomorphisms of the unit interval can be written as a union of a meager and a Haar null set.

We also examine the group of unitary transformations of the separable Hilbert space. We give a partial description of Haar positive conjugacy classes by finding a countable set of conjugacy classes and proving that every conjugacy class not contained in it is Haar null.

Essential spectra of weighted composition operators on $C(K)$

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We discuss the connection between essential spectra of weighted composition operators on $C(K)$ induced by open surjections and topological properties of the corresponding surjections.

[1] The author was supported by the travel fund of the Community College of Philadelphia

On the convergence of multiple correlation sequences with integer part polynomial iterates

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In this talk we will discus well known and new results on multiple correlation sequences. More specifically, we will show that multiple ergodic averages with iterates given by the integer part of real valued polynomials converge in the (uniform) mean. Furthermore, we show that under certain assumptions the limit is zero. Also, by the same method, which is a transference principle that enables one to deduce results for $\mathbb{Z}$-actions from results for flows, we show that the parameters in the multidimensional Szemerédi theorem for closest integer polynomials have non-empty intersection with the set of shifted primes $\mathbb{P}-1$ (or similarly of $\mathbb{P}+1$). Using the Furstenberg Correspondence Principle, we show this result by recasting it as a polynomial multiple recurrence result in measure ergodic theory.

Homogeneous spaces as coset spaces of groups from special classes

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Let X be a coset space of some topological group. Can X be a coset space of a topological group from some class ((metrizable) compact, Polish, ω-narrow, ω-balanced, etc.)?

From results of R. Arens and E. Effros, G. Ungar deduced that any homogeneous compact metrizable space is a coset space of a Polish group. From results of R. Arens and L. Kristensen it follows that metrizable compact space is a coset space of a compact metrizable group iff it is metrically homogeneous. N. Okromeshko showed that the class of coset spaces of compact groups coincides with the class of isometrically homogeneous compacta. J. van Mill proved that separable metrizable and Polish SLH spaces are coset spaces of separable metrizable and Polish groups respectively.

The results when a coset space is a coset space of ω-narrow or ω-balanced group will be presented.

[1] The author was supported by the RFBR Grant 15-01-05369

On κ-metrizable spaces

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The concept of a κ-metrizable spaces was introduced by E. Shchepin in 1976. Let $RC(X)$ denote the set of all regular closed sets of a topological space X. A topological space X is *κ-metrizable* if there exists a function $\rho : X \times RC(X) \to [0, \infty)$ satisfying the following conditions:

1. $\rho(x, C) = 0$ if and only if $x \in C$ for every $x \in X$,

2. If $C \subseteq D$, then $\rho(x, C) \geq \rho(x, D)$ for every $x \in X$,

3. $\rho(\cdot, C)$ is a continuous function for every $x \in X$,

4. $\rho(x, cl(\bigcup_{\alpha<\lambda} C_\alpha)) = \inf_{\alpha<\lambda} \rho(x, C_\alpha)$ for every non-decreasing totally ordered sequence $\{C_\alpha : \alpha < \lambda\} \subset RC(X)$ and every $x \in X$.

We say that ρ is *κ-metric* if it satisfies condition $(1) - (4)$. If ρ fulfills condition $(1) - (3)$ and $\rho(x, cl(\bigcup_{n<\omega} C_n)) = \inf_{n<\omega} \rho(x, C_n)$ for any chain $\{C_n : n < \omega\} \subset RC(X)$ and any $x \in X$, then we say that ρ is *countable κ-metric*. If a space X has a countable κ-metric, then we call this space *countably κ-metrizable*.

We show that κ-metrizable spaces is a proper subclass of countable κ-metrizable spaces. On the other hand, for pseudocompact spaces the new class coincides with κ-metrizable spaces. We prove a generalization of Chigogidze result that Čech–Stone compactification of pseudocompact countable κ-metrizable space is κ-metrizable. We also give a new characterization of existence measurable cardinal using countable κ-metric.

Combinatorics of spoke systems for Fréchet–Urysohn points

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A *spoke* for a point x in a space X is a subspace S containing $N_x = \bigcap \mathcal{N}_x = \{y \in X : x \in \overline{\{y\}}\}$ such that x has a well-ordered (by $\supseteq$) neighbourhood base with respect to S. A *spoke system* is a collection of spokes $\mathfrak{S}$ such that

$$\left\{ \bigcup_{S \in \mathfrak{S}} U_S : (U_S)_{S \in \mathfrak{S}} \text{ is a selection from } (\mathcal{N}_x^S)_{S \in \mathfrak{S}} \right\}$$

is a neighbourhood base of x with respect to X.

I introduced the structure of spoke systems in [1] and showed that their existence characterised radial points, a transfinite generalisation of the Fréchet–Urysohn property.

In this talk, I will demonstrate how certain strengthenings of the Fréchet–Urysohn property (such as the α_i-properties from [2]) correspond to productive and combinatorial properties of *almost-independent* spoke systems.

[1] R. Leek, *An internal characterization of radiality*, Topology and its Applications **177** (2014), 10–22

[2] A. V. Arhangel′skiĭ, *The frequency spectrum of a topological space and the product operation*, Transactions of the Moscow Mathematical Society **1981** (1981), no. 2, 163–200

$\mathfrak{G}$-bases in free objects over uniform spaces

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Denote by ω^ω the set of natural sequences endowed with the partial order: $f \le g$ iff $f(n) \le g(n)$ for all $n \in \omega$. We say that a uniform space X has a $\mathfrak{G}$-*base* if its uniformity $\mathcal{U}(X)$ admits a base of entourages $(U_\alpha)_{\alpha\in\omega^\omega}$ such that $U_\beta \subset U_\alpha$ for all elements $\alpha \le \beta$ in ω^ω. By $C_u(X)$ we denote the space of all uniformly continuous real-valued functions on X endowed with the pointwise partial order.

Theorem *The free locally convex space $L_u(X)$ of a uniform space X has a local $\mathfrak{G}$-base if and only if the uniformity $\mathcal{U}(X)$ of X has a $\mathfrak{G}$-base and the poset $C_u(X)$ is ω^ω-dominated.*

Similar sufficient conditions are found which imply that the free linear topological space $V_u(X)$ of a uniform space X has a local $\mathfrak{G}$-base.

Theorem *If the free locally convex space $L(X)$ of a k-space X has a local $\mathfrak{G}$-base, then the double function space with the compact-open topology $C_k(C_k(X))$ has a local $\mathfrak{G}$-base, too.*

The talk is based on the results of preprints [1], [2], [3].

[1] A. Leiderman, V. Pestov, and A. Tomita, *On topological groups admitting a base at identity indexed with ω^ω*, preprint (2015)
[2] T. Banakh and A. Leiderman, *$\mathfrak{G}$-bases in free (locally convex) linear topological spaces*, preprint (2016a)
[3] T. Banakh and A. Leiderman, *$\mathfrak{G}$-bases in free (Abelian) topological groups*, preprint (2016b)

Notes on free (Abelian) topological groups

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In this talk, I present some results on k-space and Fréchet–Uryshon property of free topological groups and free Abelian topological groups. Some questions are posed.

Theorem *Let X be a topological space in which the closure of a bounded subset in X is compact. If $F_5(X)$ is Fréchet–Urysohn, then X is compact or discrete.*

Theorem *Let X be a non-metrizable, Lašnev space. Then the following are equivalent.*

1. *$A(X)$ is a k-space.*
2. *$A_n(X)$ is a k-space for each n.*
3. *$A_4(X)$ is a k-space.*
4. *X is a topological sum of a k-space with a countable k-network consisting of compact subsets and a discrete space.*

Theorem *Assume $\mathfrak{b} = \omega_1$. For a non-metrizable Lašnev spaces X, $A_3(X)$ is a k-space if and only if $A(X)$ is a k-space.*

Theorem *Assume $\mathfrak{b} > \omega_1$. There exists a non-metrizable Lašnev space X such that $A_3(X)$ is a k-space but $A(X)$ is not.*

On group-valued continuous functions: k-groups and reflexivity

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This talk concerns two results about $\mathcal{C}(X, A)$, the set of continuous maps on a space X with values in a topological group A, equipped with pointwise operations and the compact-open topology.

Definition. A topological group G is a *k-group* if every group homomorphism $\varphi : G \to H$ into a topological group H such that $\varphi_{|K}$ is continuous for every compact subset K of G is actually continuous.

Definition. For an Abelian topological group G, let $\hat{G}$ denote the group of all continuous characters of G, and equip $\hat{G}$ with the compact-open topology. The group G is *reflexive* if the evaluation map $\alpha_G : G \to \hat{\hat{G}}$ is a topological isomorphism.

Theorem *If X is a compact Hausdorff space such that $\mathcal{C}(X, \mathbb{R}/\mathbb{Z})$ is divisible and A is a locally compact Abelian group, then:*

1. *$\mathcal{C}(X, A)$ is a k-group; and*
2. *$\mathcal{C}(X, A)$ is reflexive.*

Hereditary Interval Algebras and Cardinal Invariants

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An *interval algebra* is a Boolean algebra which is generated by a linearly order set (relative to the Boolean order). This class is not closed under taking substructures. In fact, it was shown by Nikiel, Purisch and Treybig (and independently by Odintsov) that there is a σ-centered interval algebra of cardinality $\mathfrak{c}$ which is not hereditary. On the other hand, Bekkali and Todorcevic proved that σ-centered subalgebras of interval algebras of cardinality less than $\mathfrak{b}$ are interval themselves. We show that $\mathfrak{b}$ is the minimal cardinal of a σ-centered interval algebra which is not hereditary.

[1] The author was partially supported by the grant Fondecyt Iniciacion No. 11130490

Laminations of the Unit Disk and Cubic Julia Sets

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Laminations of the unit disk were introduced by William Thurston as a topological/combinatorial model for understanding the (connected) Julia sets of polynomials, with focus on quadratic (degree $d = 2$) polynomials. This understanding can be extended, at least partially, to laminations corresponding to connected Julia sets for degree $d \geq 3$ polynomials; the Julia set is the monotone image of the lamination with semiconjugate dynamics. We focus on laminations of degree 3, and in particular, on degree 3 laminations that contain an *identity return* leaf or triangle. In the case of unicritical degree 3 polynomials, a connection between parameter spaces (laminational and analytic) is asserted.

[1] With thanks for support by the University of Alabama at Birmingham, Department of Mathematics.

Turning ternary relations into antisymmetric betweenness relations

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Let $\mathscr{R}$ be a family of nonempty subsets of a set X such that

1. all singleton subsets of X are in $\mathscr{R}$, and
2. for any a, b in X, there is $R \in \mathscr{R}$ with $a, b \in R$.

A ternary relation then arises naturally on X from such a family by writing $[a, b, c]$ (and saying *b is between a and c*) if and only if $b \in R$ for each $R \in \mathscr{R}$ with $a, c \in R$. This primitive notion of betweenness was introduced by Bankston in 2013. He showed in particular that such relations, called R-relations, are first-order axiomatizable.

An R-relation is said to be antisymmetric if $[a, b, c]$ and $[a, c, b]$ together imply $b = c$. We construct the antisymmetric closure of an R-relation and expose it as a reflector between complete lattices and their distributive counterparts.

Automorphisms of $\mathcal{P}(\lambda)/\mathcal{I}_\kappa$

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It has long been known that, consistently, there exists an automorphism of $\mathcal{P}(\mathbb{N})/fin$ which is not induced by a function $\mathbb{N} \to \mathbb{N}$. However, several questions about similar structures on uncountable cardinals remain open, for instance:

Question *Is it consistent with ZFC that there exists an automorphism of $\mathcal{P}(\omega_1)/ctble$ which is not induced by a function $\omega_1 \to \omega_1$?*

In this talk I study automorphisms of the Boolean algebra $\mathcal{P}(\lambda)/\mathcal{I}_\kappa$, where $\mathcal{I}_\kappa$ denotes the ideal of sets with cardinality less than κ, for various choices of κ and λ. I will demonstrate several conditions that imply that such an automorphism is induced by a function from λ to λ. These results will reveal connections with several classic topics from set-theoretic topology, including Q-sets, ladder systems, and Turzanski's Problem (also known as the Katowice Problem), which asks:

Question *Is it consistent with ZFC that $\mathcal{P}(\mathbb{N})/fin$ and $\mathcal{P}(\omega_1)/fin$ are isomorphic?*

Resolvable-measurable mappings of metrizable spaces

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In an old question Lusin asked when a Borel mapping $f : X \to Y$ can be decomposed into continuous mappings $f_n : X_n \to Y$, where $\{X_n : n \in \omega\}$ is a cover of X and the restriction $f|X_n = f_n$. The first affirmative answer was given by J.E. Jayne and C.A. Rogers. Recall that a mapping $f : X \to Y$ is *piecewise continuous* if X can be covered by a sequence $X_0, X_1, \ldots$ of closed sets such that the restriction $f|X_n$ is continuous for every $n \in \omega$.

Theorem *Jayne–Rogers, 1982 If X is an absolutely Souslin-$\mathcal{F}$ metrizable space and Y is a metrizable space, then $f : X \to Y$ is $\mathbf{\Delta}^0_2$-measurable if and only if it is piecewise continuous.*

This theorem was generalized a number of different ways. We give a similar statement for a metrizable completely Baire space X. Some related results will be discussed.

A mapping $f : X \to Y$ is said to be *resolvable-measurable* if $f^{-1}(U)$ is a resolvable subset of X for every open set $U \subset Y$.

Theorem *Let $f : X \to Y$ be a mapping of a metrizable completely Baire space X to a regular space Y. Then f is resolvable-measurable if and only if it is piecewise continuous.*

Ideal convergence of nets of functions with values in uniform spaces

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We consider the pointwise, uniform, quasi-uniform, and the almost uniform $\mathcal{I}$-convergence for a net $(f_d)_{d\in D}$ of functions from a topological space X into a uniform space Y, where $\mathcal{I}$ is an ideal on D. The purpose of this talk is to provide ideal versions of some classical results and to extend these to the nets of functions with values in uniform spaces. In particular, we define the notion of $\mathcal{I}$-equicontinuous family of functions on which pointwise and uniform $\mathcal{I}$-convergence coincide on compact sets. Generalizing the theorem of Arzelà, we give a necessary and sufficient condition for a net of continuous functions from a compact space into a uniform space to $\mathcal{I}$-converge pointwise to a continuous function.

The Samuel Realcompactification

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In this talk we will introduce a realcompactification for any metric space (X, d), defined as the smallest realcompactification of X such that every real-valued uniformly continuous function can be continuously extended to it. It is called the *Samuel realcompactification* according to the well known Samuel compactification associated to the family of all the bounded real-valued uniformly continuous functions. Similarly, the Lipschitz realcompactification of (X, d) is defined as the smallest realcompactification of X such that every real-valued Lipschitz function can be continuously extended to it.

We will start showing how the Samuel realcompactification of (X, d) can be described in terms of the Lipschitz realcompactification of (X, ρ_λ) for a certain family of metrics $\{\rho_\lambda\}_\lambda$ all of them uniformly equivalent to d. This description will allow to deduce when the Samuel realcompactification of (X, d) is equivalent to the Lipschitz realcompactification of (X, ρ) for some metric ρ uniformly equivalent to d. This is in fact equivalent to a problem studied by J. Hejcman in the setting of metrizable bornologies. Next, we will give a Katetov-Shirota type theorem asserting that a metric space (X, d) is Samuel realcompact if and only if X satisfies a strong property of completeness, called Bourbaki-completeness (defined by the authors), and every uniformly discrete closed subspace of X has non-measurable cardinal. In particular, this result gives an answer, in the frame of metric spaces, to a question posed by Hušek and Pulgarín.

[1] Both authors were partially supported by the MINECO Project MTM2012-34341 (Spain).

A structured construction of locally compact spaces by induction

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An archetypal example of constructing spaces by induction was given by Ostaszewski in constructing his S-space under the assumption of $\diamondsuit$. This talk introduces a much more structured type of induction in showing:

Theorem *Every stationary, co-stationary subset E of ω_1 has a locally compact, normal, quasi-perfect preimage X of cardinality $\mathfrak{b}$.*

Assuming wo l o g that the successor ordinals are the isolated points of E, let E_0 be the set of points of E which are not in the closure of $\omega_1 \setminus E$. The underlying set for X is $E_0 \cup [(E \setminus E_0) \times \mathfrak{b}]$.

The neighborhoods of $(\alpha, \xi) \in [(E \setminus E_0) \times \mathfrak{b}]$ hit an array of clopen sets defined in earlier stages in several precise ways with the use of a family of increasing functions, $f_\eta : \omega \to \omega$ $(\eta < \mathfrak{b})$ that are well-ordered and unbounded in the eventual domination order $<^*$.

On quasi-uniform box product

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It is well-known that uniform box product of a uniform space is a topology that sits between the product topology and the box product topology ([1]). In this talk we discuss the quasi-uniform box product of a quasi-uniform space.

[1] J. R. Bell, *The uniform box product*, Proc. Amer. Math. Soc. **142** (2014), no. 6, 2161–2171

1 The author was partially supported by the South African NRF
2 The author was partially supported by FRC-NWU-MC

On the shadowing property and odometers

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When we investigate the space of invariant measures from ergodic theory point of view, we are usually not that much interested in the topological structure of underlying space. By famous Jewett–Krieger theorem, we can view invariant measures as supported on minimal systems and numerous further generalizations allow to add even more topological (dynamical) properties to the underlying system. On the other hand, there are examples of systems with quite rich dynamical structure (e.g. topologically mixing) but not that much interesting invariant measures (e.g. only trivial measure, only atomic measures, etc.). In other words, connections between topology and ergodic theory (on compact metric spaces) is not that tight.

In this talk we will provide some characterizations of invariant measures in the case when a dynamical system (X, T) has the shadowing property. We will show that often invariant measures can be approximated by a special class of minimal dynamical systems. We will also comment on possibilities of approximation of entropy.

Crowded pseudocompact Tychonoff spaces of cellularity at most the continuum are resolvable

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It was a problem established by Comfort and García-Ferreira if every crowded pseudocompact space must be resolvable. Remember that every Tychonoff pseudocompact space is Baire and the existence of a crowded Baire irresolvable space is equiconsistent with the existence of a measurable cardinal. In 2014 van Mill proved that every Tychonoff, crowded, pseudocompact c.c.c. space is $\mathfrak{c}$-resolvable. In this talk we will show that every Tychonoff, crowded, pseudocompact space with cellularity at most $\mathfrak{c}$ is resolvable.

[1] The author was supported by FAPESP (Brazil) Proc. 2014/16955-2
[2] The author was partially supported by CNPq (Brazil) 307130/2013-4 and 483734/2013-6

Characterizing Noetherian spaces as Δ^0_2-analogue to compact spaces

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In the presence of suitable power spaces, compactness of $\mathbf{X}$ can be characterized as the singleton $\{\emptyset\}$ being open in $\mathcal{A}(\mathbf{X})$. Equivalently, this means that universal quantification over a compact space preserves open predicates.

Using the language of represented spaces, one can make sense of notions such as a Σ^0_2-subset of the space of Σ^0_2-subsets of a given space [1]. This suggests higher-order analogues to compactness: We can, e.g. , investigate the spaces $\mathbf{X}$ where $\{\emptyset\}$ is a Δ^0_2-subset of the space of Δ^0_2-subsets of $\mathbf{X}$. Call this notion Δ^0_2-compactness. As Δ^0_2 is self-dual, we find that both universal and existential quantifier over Δ^0_2-compact spaces preserve open predicates.

Recall that a space is called Noetherian iff every subset is compact. Within the setting of Quasi-Polish spaces [2], we can fully characterize the Δ^0_2-compact spaces. Note that the restriction to Quasi-Polish spaces is sufficiently general to include plenty of examples.

Theorem *A Quasi-Polish space is Noetherian iff it is Δ^0_2-compact.*

[1] A. Pauly and M. de Brecht, *Towards synthetic descriptive set theory: An instantiation with represented spaces*, arXiv 1307.1850.

[2] M. de Brecht, *Quasi-Polish spaces*, Annals of Pure and Applied Logic **164** (2013), no. 3, 354–381

Hereditary coreflective subcategories in categories of semitopological groups

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Let **A** be an epireflective subcategory of the category **STopGr** of all semitopological groups. It is well known that a coreflective subcategory of **A** is closed under the formation of coproducts and it is closed under the formation of extremal quotients if and only if it is monocoreflective. It is interesting to investigate the closedness of coreflective subcategories under other basic constructions. Productive coreflective subcategories were studied e.g. in [1], [2]. But little is known about hereditary coreflective subcategories.

We will present a description of the hereditary coreflective hull of a subcategory in **A**. Then we will focus on bicoreflective subcategories. It is easy to see that every hereditary coreflective subcategory of **A** is monocoreflective. We show that every hereditary coreflective subcategory that contains a group with a non-indiscrete topology is also bicoreflective in the categories **STopGr** and **QTopGr** (the category of all quasitopological groups). The situation is more complicated in other epireflective subcategories **A** of **STopGr**. We will present various sufficient conditions for a hereditary coreflective subcategory to be bicoreflective in **A**.

[1] H. Herrlich and M. Hušek, *Productivity of coreflective classes of topological groups*, Commentationes Mathematicae Universitatis Carolinae **40** (1999), no. 3, 551–560

[2] B. Batíková and M. Hušek, *Productivity numbers in paratopological groups*, Topology and its Applications **193** (2015), 167–174

Normal spanning trees in uncountable graphs, and almost disjoint families

Max F. Pitz

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In a paper from 2001 (JLMS), Diestel and Leader characterised uncountable graphs that admit a normal spanning tree through a class of forbidden graph-theoretic substructures. These substructures are strongly related to Aronszajn trees, almost disjoint families and ultrafilters.

In this talk we investigate under which circumstances these forbidden substructures can be made nice. We will see that there is a nice solution to this problem under Martin's Axiom+$\neg$CH. However, under CH or $\diamondsuit$, it seems as if we have to allow for many inequivalent forbidden substructures—but so far we only know two inequivalent classes through the work of Diestel and Leader.

I will describe the problems that are involved, draw some connections to topology and the space of ultrafilters $\beta\omega$, and explore parallels to the work of Roitman and Soukup (Fundamenta, 1998) on certain structural properties of almost disjoint families.

This is work in progress, joint with Nathan Bowler and Stefan Geschke.

On the center of distances

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Given a metric space X with the distance d, then the set

$$S(X) = \{\alpha : \forall_{x\in X} \exists_{y\in X} d(x,y) = \alpha\}$$

is called the *center of distances* of X. This notion occurs naturally in the following generalization of the theorem by J. von Neumann: *Suppose that sequences $\{a_n\}_{n\in\omega}$ and $\{b_n\}_{n\in\omega}$ have the same set of cluster points $C \subseteq X$, where (X,d) is a compact metric space. If $\alpha \in S(C)$, then there exists a permutation $\pi : \omega \to \omega$ such that $\lim_{n\to+\infty} d(a_n, b_{\pi(n)}) = \alpha$.* Also, it is used to study sets of subsums of some sequences of positive reals, as well to some impossibility proofs. We compute the center of distances of the Cantorval $\mathbb{X}$, which is the set of subsums of the sequence $\frac{3}{4}, \frac{1}{2}, \ldots, \frac{3}{4^n}, \frac{2}{4^n}, \ldots$, and also for some related subsets of the reals. Some of our results are: *If $q > 2$ and $a \geqslant 0$, then the center of distances of the set of subsums of a geometric sequence $\{\frac{a}{q^n}\}_{n\geqslant 1}$ consists of the terms $0, \frac{a}{q}, \frac{a}{q^2}, \ldots$; The center of distances of the Cantorval $\mathbb{X}$ consists of the terms $0, \frac{3}{4}, \frac{1}{2}, \ldots, \frac{3}{4^n}, \frac{2}{4^n}, \ldots$; The center of distances of the set $[0, \frac{5}{3}] \setminus$ Int $\mathbb{X}$ is trivial, since it consists of only Zero; Neither $[0, \frac{5}{3}] \setminus$ Int $\mathbb{X}$ nor $\mathbb{X} \setminus$ Int $\mathbb{X}$ is the set of subsums of a sequence; The center of distances of the set $\mathbb{X} \setminus$ Int $\mathbb{X}$ consists of the terms $0, \frac{1}{4}, \frac{1}{16}, \ldots, \frac{1}{4^n}, \ldots$; Let $A \subset \{\frac{2}{4^n} : n > 0\} \cup \{\frac{3}{4^n} : n > 0\} = B$ be such that $B \setminus A$ and A are infinite. Then the set of subsums of a sequence consisting of different elements of A is homeomorphic to the Cantor set.*

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On lineability of classes of functions with various degrees of (dis)continuity

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We will provide a short overview of recent research on the existence of "large" algebraic structures (e.g., vector spaces, groups) within various classes of real functions. Our focus will be on the classes of functions with various degrees of continuity (discontinuity): almost continuous functions (AC), Darboux functions, Sierpiński–Zygmund functions (SZ). We will prove the following theorem.

Theorem *Assuming the Continuum Hypothesis, the intersection of the classes of almost continuous functions* (AC) *and Sierpiński-Zygmund functions* (SZ) *contains an additive semigroup of size* $2^{\mathfrak{c}}$.

In addition, we will show that it is consistent with ZFC that the cardinality of the "largest" vector space within the class of Sierpiński–Zygmund functions (SZ) is equal to the cardinality of the "largest" vector space within the class of almost continuous Sierpiński–Zygmund functions (AC ∩ SZ). Namely, we show the following result.

Theorem *It is consistent with ZFC that* $\mathcal{L}(\mathrm{AC}\cap\mathrm{SZ}) = \mathcal{L}(\mathrm{SZ}) = (2^{\mathfrak{c}})^+$. (*where* $\mathcal{L}(F)$ *is called the lineability of* $F \subseteq \mathbb{R}^{\mathbb{R}}$ *and is defined as* $\min\{\kappa : F \cup \{0\}$ *doesn't contain a vector space of dim* $\kappa\}$)

It is unclear if the two cardinal numbers from the above theorem can be different under the assumption of the existence of an almost continuous Sierpiński-Zygmund function.

Question *Is it consistent with ZFC that* $\mathrm{AC} \cap \mathrm{SZ} \neq \emptyset$ *and* $\mathcal{L}(\mathrm{AC} \cap \mathrm{SZ}) < \mathcal{L}(\mathrm{SZ})$?

Embedding cartesian products of graphs in symmetric products

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Let X be a metric continuum. We denote by X^n its cartesian product and by $F_n(X)$ its symmetric product, i.e. the hyperspace of all non-empty subsets of X with at most n points, $F_n(X)$ endowed with the Hausdorff metric. We will consider some conditions about the ramification points to ensure that we get an embedding $X^n \to F_n(X)$. In this context it is possible to characterize the arc.

[1] The author was partially supported by the PROFOCIES and DAAD

Equilibrium under uncertainty with Sugeno payoff

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Dow and Werlang considered two players game where belief of each player about a choice of the strategy by the other player is a non-additive measure called capacity and expected payoff function is defined by Choquet integral. They introduced some equilibrium notion for such games and proved its existence.

An alternative to so-called Choquet expected utility model is the qualitative decision theory. The corresponding expected utility is expressed by Sugeno integral. This approach was widely studied in the last decade.

We consider the abovementioned equilibrium notion for a game with expected payoff function defined by Sugeno integral. We prove the existence of equilibrium under uncertainty which is expressed by possibility (or necessity) capacities. Since the spaces of possibility and necessity capacities have no natural linear convex structure, we use some non-linear convexity.

Extremally disconnected topological groups

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In this talk we present our partial results on the well-know problem of Arhangel'skii on the existence in ZFC of a non-discrete Hausdorff extremally disconnected (or ED for short) topological group; it was posed in 1967 and has been extensively studied since then. The problem is still open even for separable (eq., countable) groups, although several consistent examples have been constructed.

We have two lines of research:

1. (with M. Hrušák) The connection between the existence of a separable non-discrete Hausdorff ED topological group and the existence of a nowhere dense ultrafilter on ω.

2. We introduce the notion of algebraic free sequence on topological group and we show that if an ED Boolean topological group admits a countable nontrivial algebraic free sequence, then there is a rapid ultrafilter on ω.

[1] The author was partially supported by the PAPIIT grant IN108014 and CONACyT grant 177758.

Cauchy action of groups

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A Cauchy group $(G, D, \cdot)$ has a Cauchy action on a filterspace (X, C), if it acts in a compatible manner. This simple additional property of a group action leads to a vast area of research in many directions with countless categorical implications. The properties of this action, such as transitiveness and its compatibility with various modifications of (X, C) are explored in this paper. In addition, any Cauchy action can have a standard extension to a completion of the G-space. The primary goal here is to establish the G-invariance of the extension of a Cauchy map to a complete G-space.

A word or two must be noted about the origin and scope of this work. Motivated by the wide range of applications of group action, convergence-group action was introduced with its special impact on homeomorphism groups ([1]), and later it led to several research articles (see, for example, [2], [3]). The current paper opens a new dimension to the applications of group action by establishing a close interaction between Cauchy action and continuous actions.

The notion of Cauchy continuity in group action is a natural blend of set-theoretic topology and algebra. In a nutshell, this is a minuscule attempt by the author, which very well promotes some avenues for an interesting area of research in general topology.

[1] N. Rath, *Action of convergence groups*, Topology Proceedings **27** (2003), no. 2, 601–612.

[2] E. Colebunders et al., *Convergence approach spaces:Actions*, Applied Categorical Structures **24** (2016), no. 2, 147–161.

[3] H. Boustique et al., *Convergence semigroup actions: generalized quotients*, Applied general Topology **10** (2009), no. 2, 173–186.

Maximal Homogeneous Spaces

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We say that a space X is *maximal homogeneous* if X is a maximal homogeneous subspace of βX containing X. For any homogeneous space X, there exists a unique maximal homogeneous space $H(X) \subset \beta X$ for which $X \subset H(X) \subset \beta X$. For example, any first countable homogeneous space is maximal homogeneous. Let p be a free ultrafilter on ω. We say that a space X *totally countably p-compact* if, for any infinite $M \subset X$, there exists an infinite $L \subset M$ such that any sequence $(x_n)_{n\in\omega} \subset L$ ($x_i \neq x_j$ for $i \neq j$) has a p-limit in X. Any totally countably compact space (in particular, any sequentially compact space) is totally countably p-compact for any $p \in \omega^* = \beta\omega \setminus \omega$. Clearly, any totally countably p-compact space is contable compact.

Theorem *If $p \in \omega^*$ and X is totally countably p-compact space, then X^ω is totally countably p-compact and, hence, countably compact.*

Theorem *Let X be a maximal homogeneous extremally disconnected space. If X contains a nonclosed dicrete sequence of points, then X is totally countably p-compact for some $p \in \omega^*$.*

Note that all known examples of homogeneous extremally disconnected countably compact spaces are maximally homogeneous.

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Recovering a Compact Hausdorff Space X from the Compatibility Ordering on $C(X)$

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Let X and Y be compact Hausdorff spaces. Let $f, g \in C(X)$ where $C(X)$ denotes the space of continuous functions on X. We say that g dominates f in the compatibility ordering if g coincides with f on the support of f. Our main result states that X and Y are homeomorphic if and only if there exists a compatibility isomorphism $T : C(X) \to C(Y)$. We derive several classical theorems of functional analysis as easy corollaries to our result:

If X and Y are compact Hausdorff spaces, we obtain that they are homeomorphic provided that there exists a bijection $T : C(X) \to C(Y)$ satisfying one of the following conditions:

1. T is a ring isomorphism (Gelfand–Kolmogorov);

2. T is multiplicative (Milgram);

3. T the ordinary pointwise ordering (Kaplansky);

4. $Tf \cdot Tg = 0$ whenever $f \cdot g = 0$ (Jarosz).

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Fuzzy uniform structures

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It is well-known that, although not every uniformity is metrizable, every entourage of a uniformity belongs to a coarser uniformity generated by a certain pseudometric. This allows to identify a uniformity with a family of pseudometrics called a *gauge* or a *uniform structure*. In 2010, Gutiérrez García, S. Romaguera and M. Sanchis [1] proved that the above is also true when you consider fuzzy pseudometrics, i.e. every uniformity is, categorically speaking, equivalent to a family of fuzzy pseudometrics satisfying certain properties.

On the other hand, we can find essentially different notions of uniformity in fuzzy topology. In our talk we will show how probabilistic uniformities as well as Lowen uniformities are equivalent, categorically speaking, to certain families of fuzzy pseudometrics. This allows for clarifying the relationship between classical uniformities and these types of fuzzy uniformities.

[1] J. Gutiérrez-García, S. Romaguera, and M. Sanchis, *Fuzzy uniform structures and continuous t-norms*, Fuzzy Sets Syst. **161** (2010), no. 7, 1011-1021

[1] The author acknowledges the support of the Ministry of Economy and Competitiveness of Spain under grant MTM2015-64373-P

On Corson and Valdivia compact spaces

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The classes of Corson and Valdivia compact spaces have been object of intensive study in Topology and Functional Analysis. These classes of compact spaces have several nice properties and are interesting in many respects. A very interesting characterization of Valdivia compact spaces was obtained by Kubiś and Michalewski in 2006; a compact space is Valdivia if and only if admits a commutative r-skeleton. M. Cúth proved in 2014 that a compact space is Corson if and only if has a full r-skeleton. We present one proof for these two results, in the spirit of Bandlow's characterization of Corson compact spaces. Besides, we establish another characterization of Valdivia compact spaces by using a monotone structure of retractions and networks. It happens that, in general, this last structure has stronger properties than r-skeletons. We also obtain a characterization of $C_p(X)$ for a Corson compact space X trough a monotone structure of $\mathbb{R}$-quotient maps. Using that result and some duality theorems, we provide a new internal characterization of Corson compact spaces trough monotone families of closed and open sets. Some applications of these results are presented.

Star P and weakly star P properties

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For a topological property P, we say that a space X is *star P* (respectively, *weakly star P*) if for every open cover $\mathcal{U}$ of the space X, there is a subspace A of X satisfying P such that

$$X = st\,(A, \mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$$

(respectively, $X = cl_X st\,(A, \mathcal{U})$).

In this talk we focus our attention on the star *ccc* and the star countable spread properties. We give the relations between these two properties and the well-known star countable and star Lindelöf properties. Under CH we give an example of a Tychonoff star countable spread space which is not star countable. We also investigate the weakly star countable spaces and weakly star Lindelöf spaces. In particular we show that if the hyperspace $F[X]$ of all non-empty finite subsets of X endowed with the Pixley–Roy topology where X has countable tightness is weakly star Lindelöf, then X is hereditarily Lindelöf and hereditarily separable.

Examples of absorbers in continuum theory

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The theory of absorbing sets, developed in the eighties and nineties of the last century, enables to establish that some incomplete metric spaces are homeomorphic to certain standard objects. Absorbers in a given Borel or projective classes are topologically unique and they have their models contained in the Hilbert cube I^ω. Furthermore, some families of compact subsets of Euclidean spaces (e.g. ANR's, locally connected continua, compacta with nonempty interiors, compact countable subsets of $[0,1]$) were recognized as absorbers in appropriate Borel or projective classes.

Looking for another natural absorbers in continuum theory we focus on subspaces of 2^X and $C(X)$, where X is a compact connected manifold (with or without boundary). We give several new examples of hyperspaces being F_σ-, $F_{\sigma\delta}$-, Π^1_1- and $D_2(F_\sigma)$-absorbers ($D_2(F_\sigma)$ denotes the small Borel class consisting of differences of two F_σ-sets).

Selectively sequentially pseudocompact group topologies on Abelian groups

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We call a space X *selectively sequentially pseudocompact* if for each countable family $\{U_n : n \in \mathbb{N}\}$ of non-empty open subsets of X one can choose a point $x_n \in U_n$ for every $n \in \mathbb{N}$ in such a way that the sequence $\{x_n : n \in \mathbb{N}\}$ has a convergent subsequence. Clearly, sequentially compact → selectively sequentially pseudocompact → strong pseudocompact → pseudocompact. Unlike the strong pseudocompactness property of S. García-Ferreira and Y. F. Ortiz-Castillo whose preservation under products even in topological groups remains an open question, the class of selectively sequentially pseudocompact spaces is closed under taking arbitrary products.

We study the following general question: *If an Abelian group admits a pseudocompact group topology, does it also admit a selectively sequentially pseudocompact group topology?* Under the Singular Cardinal Hypothesis (SCH), we provide a positive answer to this question for the following classes of Abelian groups:

1. torsion groups;
2. torsion-free groups;
3. $\mathcal{V}$-free groups in any variety $\mathcal{V}$ of Abelian groups.

This provides a partial answer to a question of S. García-Ferreira and A. H. Tomita (with strong pseudocompactness strengthened to selective sequential compactness).

Setwise and Pointwise Betweenness via Hyperspaces

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This work investigates the notion of setwise betweenness, a concept introduced by P. Bankston as a generalisation of pointwise betweenness. In the context of continua, we say that a subset C of a continuum X is between distinct points a and b of X if every subcontinuum K of X containing both a and b intersects C. The notion of an interval $[a, b]$ then arises naturally. Further interesting questions derive from considering such intervals within an associated hyperspace on X. We explore these ideas within the context of the Vietoris topology on the set 2^X of all nonempty closed subsets of a T_1 space X. Moreover an alternative pointwise interval, derived from setwise intervals, is introduced.

[1] The first author was partially supported by the iraqi minisitery of heigher eduactation and scientific Research

Convergence measure spaces: An approach towards the duality theory of convergence groups

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The theory of integration on topological spaces is well developed but there are a lot of situations in analysis where non-topological convergence originate. Thus, it is natural to extend the theory of abstract integration from topological spaces to convergence spaces as this kind of approach provides the basic framework for analysis on convergence spaces which are not necessarily topological.

We introduce the term convergence measure space whose underlying idea is to define the σ-algebra compatible with the convergence structure using the modification of the convergence space. Further, we present some facts about the measures on general convergence spaces obtained during the investigation of the regular measures in the realm of convergence spaces.

Using this definition we make an attempt to extend the idea of the invariant measures from topological groups to the groups with limit related structures which are not necessarily topological and finally, we point out some general questions that arise with this kind of approach in the extension of the Pontryagin duality theory of topological Abelian groups to the class of convergence groups.

[1] Pranav Sharma is supported by Junior Research Fellowship of UGC, New Delhi.

Minimality of the Semidirect Product

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A topological group is minimal if it does not admit a strictly coarser Hausdorff group topology. We prove that for a compact topological group G, the semidirect product $G \leftthreetimes P$ is minimal for every closed subgroup P of Aut(G). In general, the compactness of G is essential; $G \leftthreetimes P$ might be nonminimal even for precompact minimal groups G as it follows from an example of Eberhardt–Dierolf–Schwanengel. Some of the results were inspired by a work of Gamarnik.

Almost disjoint families and relative versions of covering properties of κ-paracompactness type

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The main goal of this work is to investigate – within the realm of Isbell–Mrówka spaces, i.e., spaces constructed from almost disjoint families – some relative versions of covering properties of κ-paracompactness type, inspired by a comprehensive list of strengthenings of countable paracompactness introduced by M. E. Rudin in [1]. For any property $\mathcal{P}$ among the ones presented, we will say that an almost disjoint family $\mathcal{A}$ satisfies $\mathcal{P}$ if it satisfies a *relative* version of $\mathcal{P}$ in the corresponding Isbell–Mrówka space. We present combinatorial characterizations of the a.d. families with some of these new relative topological properties and prove several related results; for instance, it is shown that maximal almost disjoint families are not countably paracompact. The talk finishes with a number of questions and open problems

This is a joint work with Charles Morgan (UCL, London) and Dimi Rangel (USP, Sao Paulo).

[1] M. E. Rudin, κ-Dowker spaces, in *Aspects of topology* , number 93 in London Math. Soc. Lecture Note Ser. (Cambridge Univ. Press, Cambridge), p. 175–193.

[1] The travel of the presenting author was sponsored by FAPESB/Bahia.

On Boolean topological groups

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Boolean topological groups often arise in topological algebra as examples of topological groups with exotic properties. They play the key role in the theory of extremally disconnected topological group, because, by Malykhin's 1975 theorem, any extremally disconnected group contains an open Boolean subgroup. The talk considers properties of such groups with special emphasis on free Boolean topological groups and extremally disconnected groups.

Some properties of free Boolean topological groups differ drastically from those of free and free Abelian topological groups. Thus, we show that the free Boolean topological group $B(X)$ on a space X does not always contain X^2 as a subspace (while the free and free Abelian groups on X contain all X^n as closed subspaces) and discuss other examples of such properties.

As mentioned above, Boolean topological groups play the key role in Arhangel'skii's 1967 problem on the existence of extremally disconnected topological groups. So far, only partial results have been obtained. One of results presented in the report is that any closed linearly independent subset of a countable extremally disconnected group contains at most one nonisolated point.

Finally, a natural relationship between countable Boolean topological groups and notions of forcing (in particular, Mathias and Laver forcings) is considered.

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A local Ramsey theory for block sequences

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Gowers [1] proved an approximate Ramsey theorem for analytic partitions of the space of block sequences in a Banach space. Exact, discretized, versions of this result were later given by Rosendal [2]. We isolate the combinatorial properties of the space of block sequences which enable these constructions, and prove that they can be carried out within certain subfamilies, analogous to selective coideals and the role they play in Mathias' [3] local form of Silver's theorem for analytic partitions of $[\mathbb{N}]^\infty$. We consider applications of these results to understanding the combinatorial structure of projections in Calkin algebra. Under large cardinal assumptions, these results are extended to partitions in $\mathbf{L}(\mathbb{R})$.

[1] W. Gowers, *An infinite ramsey theorem and some banach-space dichotomies*, Annals of Mathematics (2) **156** (2002), no. 3, 797-833

[2] C. Rosendal, *An exact ramsey principle for block sequences*, Collectanea Mathematica **61** (2010), no. 1, 25-36

[3] A. Mathias, *Happy families*, Annals of Mathematical Logic **12** (1977a), no. 1, 59–111

[1] The author has been partially supported by NSERC and the NSF

Infinite games and chain conditions

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We use games to prove results about cardinal invariants in topology. For example, the following theorem is related to an old problem of Arhangel'skii.

Theorem *Let X be a compact Hausdorff space such that player two has a winning strategy in the weak Rothberger game of length ω_1. Then every cover by G_δ subsets of X has a continuum-sized subcollection with a G_δ-dense union.*

And the following may be considered as a partial positive ZFC answer to Suslin's Problem.

Theorem *Let X be a compact linearly ordered space such that player two has a winning strategy in the open-open game. Then X is separable.*

[1] The author was supported by FAPESP postdoctoral grant 2013/14640-1

Baire classes of affine vector-valued functions

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We investigate Baire classes of strongly affine mappings with values in Fréchet spaces. We show, in particular, that the validity of the vector-valued Mokobodzki's result on affine functions of the first Baire class is related to the approximation property of the range space. We further extend several results known for scalar functions on Choquet simplices or on dual balls of L_1-preduals to the vector-valued case. This concerns, in particular, affine classes of strongly affine Baire mappings, the abstract Dirichlet problem and the weak Dirichlet problem for Baire mappings. Some of these results have weaker conclusions than their scalar versions. We also establish an affine version of the Jayne–Rogers selection theorem.

Topological approach to "non-topological" ultrafilters

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In the literature there are known three types of hierarchies of ultrafilters on the set of natural numbers: level ultrafilters and ordinal ultrafilters defined by J.E. Baumgartner in *Ultrafilters on* ω, 1995, and the P-hierarchy defined by the present autor *P-hierarchy on* ω, 2008. The first of them was defined by use of topological properties of the real line, two others were defined by order types and by combinatorical properties, respectively. Although these three hierarchies are (ZFC-consistently) different, all of them may be characterized in terms of level ultrafilters considered for different topological spaces. We also discuss some properties of classes these hierarchies.

G_δ covers of compact spaces

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For any space X let X_δ denote X endowed with the topology generated by all G_δ subsets of X. Arhangel'skii asked whether the weak Lindelof degree of X_δ is bound by the continuum whenever X is compact and Hausdorff. We construct an example which answers this question in the negative.

Productively (and non-productively) Menger spaces

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A topological space is *Menger* if, for every sequence of open covers, we can produce a new cover by chosing finitely many open sets from each of the given covers. Menger's property is strictly stronger than being Lindelöf. Every σ-compact space is Menger, and even *productively* so: Every product of a σ-compact space and a Menger space is Menger.

Based on weak set theoretic hypotheses, we construct, in a purely combinatorial way, Menger sets of real numbers whose product is not Menger.

The *Hurewicz property* is a strong form of Menger's property. Using our method, we prove, assuming a portion of CH, that every productively Menger space is productively Hurewicz, and that the converse implication is not provable.

[1] The first author was partially supported by Polish National Science Center UMO-2014/12/T/ST1/00627

Definable Versions of Menger's Conjecture

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Menger's conjecture that Menger spaces are σ-compact has been refuted in ZFC. However, the question of whether it holds for spaces that are "definable" in some sense remains of interest. Hurewicz proved it for completely metrizable and indeed for analytic metrizable spaces; Arhangel'skiĭ proved it for analytic spaces in general. We prove it for Čech-complete spaces. It was known to be undecidable for co-analytic sets of reals; we extend this to co-analytic topological groups. A more general class of spaces than the analytic ones are the K-analytic spaces studied by Frolík and by Rogers and Jayne. Among the many results we prove, let us mention that Menger K-analytic spaces need not be σ-compact, but that they satisfy the weaker Hurewicz property.

[1] The first author was supported by NSERC grant A-7354.
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Completeness properties on $C_p(X, Y)$ spaces

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In this talk we will present some results which have to do with the characterization of several kinds of pseudocompleteness and compactness properties in spaces of continuous functions of the form $C_p(X, Y)$. In particular, we prove that for every space X and every separable metrizable topological group G for which $C_p(X, G)$ is dense in Y^G, $C_p(X, G)$ is weakly α-favorable if and only if every countable subset N of X is discrete and C_G-embedded in X.

Moreover, we obtain two generalizations of a result that is due to V.V. Tkachuk:

Theorem *Let G be a separable completely metrizable topological group. If H is a dense subgroup of G^X and H is homeomorphic to G^Y for some set Y, then $H = G^X$.*

Theorem *Let G be a realcompact Čech-complete weakly α-favorable topological group with countable pseudocharacter and let X be regular $C^G_{<\omega}$-discrete. Then, $C_p(X, G) \cong G^\kappa$ if and only if X is a discrete space of cardinality κ.*

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The Bolzano property and the cube-like complexes

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B. Bolzano proved that if a function f continuous in a closed interval $[a, b]$ changes signs at the endpoints, i.e. $f(a) \cdot f(b) \leq 0$, then this function equals zero at one point of the interval at least. Nearly a hundred years later, H. Poincaré announced the n-dimensional version of this theorem. In 1940, C. Miranda rediscovered the Poincaré theorem and showed that it is equivalent to the Brouwer fixed point theorem.

During our talk we present a topological version of the Poincaré–Miranda theorem, called the Bolzano property. Next, we define the class of polyhedrons, called n-cube-like, which are generalization of the n-cubes. To show that they have the Bolzano property we adopt the n-dimensional version of Steinhaus chessboard theorem. Moreover, we investigate under what conditions the inverse limit preserves the Bolzano property and we give a characterization of the Bolzano property for locally connected spaces. Finally, we explain the relation between the Bolzano property and the covering dimension.

Verbal functions of a group

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If G is a group, take the symbol x as a variable, and denote by $G[x] = G * \langle x \rangle$ the free product of G and the infinite cyclic group $\langle x \rangle$. Call $G[x]$ *the group of words with coefficients in* G, and its elements w, *words in* G. An element $w \in G[x]$ has the form $w(x) = g_1 x^{\varepsilon_1} g_2 x^{\varepsilon_2} \cdots g_n x^{\varepsilon_n}$, for an integer $n \geq 1$, elements $g_1, \ldots, g_n \in G$ and $\varepsilon_1, \ldots, \varepsilon_n \in \{-1, 1\}$. A *verbal function* $G \to G$ is the associated evaluation function $f_w : G \to G$ (defined by $g \to w(g)$) of a word $w \in G[x]$. The *elementary algebraic subsets* of G, introduced by Markov in 1944, can be obtained by using verbal functions as the subsets of the form $E_w = f_w^{-1}(\{e_G\}) \subseteq G$, i.e., E_w is the solution-set of the equation $w(x) = e_G$ in G.

The family $\mathbb{E}_G$ of elementary algebraic subsets can be taken as the prebase of the closed sets of a unique T_1 topology $\mathfrak{Z}_G$ on G, called the *Zariski topology of* G; Markov called the $\mathfrak{Z}_G$-closed sets *algebraic subsets of* G. More generally, given a subset $W \subseteq G[x]$, the family $\mathcal{E}(W) = \{E_w \mid w \in W\} \subseteq \mathbb{E}_G$ gives a topology $\mathfrak{T}_W$ on G (the *partial Zariski topology* determined by W) having $\mathcal{E}(W)$ as a subbase for its closed sets. Obviously, $\mathfrak{T}_W \subseteq \mathfrak{Z}_G$ and $\mathfrak{T}_{G[x]} = \mathfrak{Z}_G$.

The *Markov topology* $\mathfrak{M}_G$ of G is the intersection of every Hausdorff group topology on G. It is a T_1 topology, but it need not be neither a group topology, nor even Hausdorff; it contains $\mathfrak{Z}_G$, so that $\mathfrak{T}_W \subseteq \mathfrak{Z}_G \subseteq \mathfrak{M}_G$ for every $W \subseteq G[x]$.

For some classes of groups G, I will describe an appropriate family $W \subseteq G[x]$ such that either $\mathfrak{T}_W = \mathfrak{Z}_G$ or $\mathfrak{T}_W = \mathfrak{Z}_G = \mathfrak{M}_G$, and I will give examples and recent results on these topics.

Dehn filling of a Hyperbolic 3-manifold

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An operation of Dehn surgery is defined for manifolds only in dimension 3 and consists of cutting a tubular neighborhood of a closed curve and gluing back a solid torus to its boundary along a surgery slope. The theorem of Lickorish-Wallace says that any closed, orientable connected 3-manifold can be obtained by Dehn surgery along a link in the 3-sphere.

Later W. Thurston proved a remarkable theorem that Dehn surgery of a cusped hyperbolic 3-manifold is again a hyperbolic manifold for all slopes except a finite set of them.

We investigate the hyperbolic Dehn surgery namely Dehn parental test. We verify if one of two manifolds is obtained from another one via hyperbolic Dehn filling using previous results of C.Hodgson-S.Kerckhoff and R.Haraway.

Companions of partially ordered sets

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The results we discuss were motivated by the (now retracted) claim by N. Howes and W. Sconyers that "every normal, linearly Lindelöf space is Lindelöf." Let $(D, \leq)$ be a partially ordered set. A well ordered set $(C, \preceq)$ is called a companion of $(D, \leq)$ provided C is a cofinal subsets of $(D, \leq)$, and $\preceq$ is a well order on C such that for every $c_1, c_2 \in C$ if $c_1 \leq c_2$ then $c_1 \preceq c_2$. The Ordering Lemma says that every partially ordered set has a companion. Given a directed set $(D, \leq)$ and a net $f : D \to X$, the restriction $f \restriction C$ of the net to the companion is a transfinite sequence. We discuss how the convergence and clustering of $f \restriction C$ is related to the convergence and clustering of f. We discuss the difference between converging and clustering of transfinite sequences. We give an example to show that a companion sequence $f \restriction C$ can have a cluster point but f has no cluster point.

Random elements of large groups: Discrete case

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The automorphism groups of countable homogeneous structures are usually interesting objects from group theoretic and set theoretic perspective. The description of typical (with respect to category) elements of such groups is a flourishing topic with a wide range of applications. A natural question is that whether there exist measure theoretic analogues of these results. An obvious obstacle in this direction is that such automorphism groups are often non-locally compact, hence there is no natural translation invariant measure on them.

Christensen introduced the notion of Haar null sets in non-locally compact Polish groups which is a well-behaved generalisation of the null ideal to such groups. Using Christensen's Haar null ideal it makes sense to consider the properties of a random element of the group. We investigate these properties, giving a full description of random elements in the case of the automorphism group of the random graph and the rational numbers (as an ordered set).

Chaos in hyperspaces of nonautonomous discrete systems

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Given a topological space X, let $f_n : X \to X$ be a continuous function for each positive integer n, and $f_\infty = (f_1, f_2, \ldots, f_n, \ldots)$. The pair (X, f_∞) denotes the *nonautonomous discrete dynamical system* (NDS, for short). Given a NDS (X, f_∞), it induces a NDS $(\mathcal{K}(X), \overline{f_\infty})$, where $\mathcal{K}(X)$ is the hyperspace of all non-empty compact subsets of X endowed with the Vietoris topology and, for every positive integer n, $\overline{f_n} : \mathcal{K}(X) \to \mathcal{K}(X)$ is the continuous function induced by f_n. We study the interaction of some dynamical properties (like transitivity, weakly mixing, points with dense orbit and density of periodic points) of a NDS (X, f_∞) and its induced NDS $(\mathcal{K}(X), \overline{f_\infty})$. Among other results, we show that $(\mathbb{I}, f_\infty)$ is weakly mixing of order 3 if and only if $(\mathcal{K}(\mathbb{I}), \overline{f_\infty})$ is weakly mixing of order 3, where $\mathbb{I} = [0, 1]$. We also present examples of NDS showing that the classical result stating that transitivity is a sufficient condition for an autonomous discrete dynamical system on the interval to be Devaney chaotic fails to be true for NDS.

[1] The author was partially supported by PROFOCIES

Base Tree Phenomenological Horizons

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Various mathematical asymptotics (like small/large, slow/fast) create phenomena of horizons we are challenged to perceive.

One such phenomenon can be observed when considering border between absolute convergence and divergence in c_0^+ ([1]). Classical tests for absolute convergence/divergence decide only sets of first category in ℓ^1 , $\ell^2 \setminus \ell^1$ resp. There is an (ω_1, ω_1^*) Hausdoff gap filling this. Comparison ordering of absolutely convergent series is upwards directed, comparison ordering of absolutely divergent series (downwards) has base tree property ([2]) and various cardinal characteristics of bottom and upper part of this horizon seem to depend on axioms/models of ZFC (e.g. equal under CH). Similar phenomenon can be observed on partitions of omega with refinement, see [3].

We show that $\mathfrak{h}\left(\bigcap_{n=1}^{\infty} \ell^{1+\frac{1}{n}} \setminus \ell^1, <^*\right)$ and $\mathfrak{h}(P(\omega)/_{fin}, \subseteq^*)$ are consistently different (see [1]). So, this horizon is behind ℓ^p hierarchy, is topologically large and set-theoretically sensitive.

We present a more general framework for (separative quotients of) horizons, recall some old and new examples, results and problems.

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Metrizable Cantor cubes that fail to be compact in some models for ZF

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The statement that every countable non-void collection of non-void finite sets has a choice function is denoted by CC(fin). It was proved by me in 2015 that it holds true in ZF that, for a set J, the Cantor cube 2^J is metrizable if and only if J is a countable union of finite sets. This implies that, in every model for ZF+¬CC(fin), there exist Cantor cubes that are simultaneously metrizable and not second-countable. I shall offer a proof to the following new result:

Theorem *It holds true in ZF that the following conditions (1)-(5) are all equivalent:*

1. *CC(fin).*
2. *Every metrizable Cantor cube is compact.*
3. *If a Cantor cube is metrizable, then its compact bornology is metrizable.*
4. *If a Cantor cube is first-countable, then its compact bornology is quasi-metrizable.*
5. *Every metrizable product of finite spaces is compact.*

Therefore, in every model for ZF+¬CC(fin), there exist metrizable Cantor cubes that are non-compact. Other consequences of it can be shown.

Borel's conjecture for the Marczewski ideal

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We will show the following ZFC theorem which is recently finished work joint with Jörg Brendle: There is no set of size continuum which is "s_0-shiftable", i.e., which can be translated away from every set in the Marczewski ideal s_0 (where a set of reals is in s_0 if for every perfect set there is a perfect subset disjoint from it).

We will concetrate on regular continuum, since the proof is easier in this case.

The theorem is very much in contrast to the respective situation when s_0 is replaced by the meager ideal: there are models (e.g., all models that satisfy CH) with large meager-shiftable (i.e., strong measure zero) sets.

Our original proof dealt with the reals in the sense of the Cantor space 2^ω. However, it can be generalized to other Polish groups.

The comparission of topologies for the fundamental group and for generalized covering spaces

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The first traces of attempts to consider the fundamental group as an object, which apart from its algebraic structure also has a topological structure, go back to the fifties, but they remained essentially without echo, till Biss in 2002 published a paper, where he revitalized this idea. The topology proposed by him was different from this inverse-limit topolopgy that had appeared in the fifties already, but although his paper contained several mistakes it must be regaraded the essential paper which revitalized this idea. Amongst the mistakes were the confusion between the fundamental and the first Čech-homotopy group of Hawaiian Earrings, and the assertion that the topology that he had proposed for the fundamental group would always induce the structure of a topological group. The assertion was believed for a couple of years, before almost simultaneously it was disproven by Fabel and Brazas. For the time being, we are aware of 5 different definitions of topologies on the fundamental group. The main part of the talk will be used for desribing the concepts and results on comparing the types of open sets that they generate. The talk will also report on similar results and attempts to extend the definition to those generalized covering spaces which are modelled on the point set $\widetilde{X} := \{w : [0,1] \to X, w(0) = x_0\}/\sim$, i.e. on a set of homotopy classes of paths which just differs from the fundamental group by not requiring that the paths return to the base point.

Some categorical aspects of coarse spaces and balleans

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Roe's coarse spaces and Protasov's balleans are known to be equivalent constructions describing large scale geometry of spaces. The talk is focused on two categories: the category **Coarse**, which has these structures as objects and bornologous maps as morphisms, and its quotient category **Coarse**$/_\sim$, where $\sim$ is the closeness relation between morphisms. We show that the category **Coarse** is topological, so that the description of its monomorphisms and epimorphisms is well known. Moreover we provide an explicit description of some categorical constructions in **Coarse** such as products, coproducts and quotients (of those the last case is quite delicate and requires particular attention). This allows us to describe, among others, the monomorphisms and the epimorphisms in **Coarse**$/_\sim$ and deduce that the category **Coarse**$/_\sim$ is balanced, since its bimorphisms are precisely the isomorphisms, namely the coarse equivalences.

Combinatorial covering properties and posets with fusion

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A topological space X has the *Hurewicz property* if for every sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of open covers of X there exists a sequence $\langle \mathcal{V}_n : n \in \omega \rangle$ such that $\mathcal{V}_n \in [\mathcal{U}_n]^{<\omega}$, and $\{n \in \omega : x \notin \cup \mathcal{V}_n\}$ is finite for all $x \in X$. If we simply require that $\{\cup \mathcal{V}_n : n \in \omega\}$ is an open cover of X then we get the definition of the *Menger property*. In our talk we shall discuss the behavior of these properties in the Sacks, Laver, and Miller models. For instance, in the Laver and Miller models metrizable spaces with the Hurewicz and Menger properties, respectively, enjoy certain form of concentration, which helps to analyze their products. In particular, the following theorem follows from a combination of results recently proven by Szewczak, Tsaban, Repovš, and myself.

Theorem *In the Laver model for the consistency of the Borel's conjecture, the product of any two Hurewicz spaces is again Hurewicz provided that it is a Lindelöf space. In particular, the product of any two Hurewicz metrizable spaces is Hurewicz in this model.*

[1] The author would like to thank the Austrian Science Fund FWF (Grant I 1209-N25)

Locally Roelcke precompact Polish groups

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We consider the property of local Roelcke precompactness. Groups with this property form a proper subclass of the locally (OB) groups, while still generalizing both the Roelcke precompact and the locally compact Polish groups. We consider examples and properties of such groups, and of a related object, the ideal of relatively Roelcke precompact subsets, for an arbitrary Polish group.

POSTERS

About 20 posters are presented at the Toposym 2016. This book contains only abstracts of posters submitted before July 10, 2016. All posters are accompanied by oral presentations.

The exponential law for partial, local and proper maps and its application to otopy theory

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The term *otopy* was introduced by J.C. Becker and D.H. Gottlieb to designate a very useful generalization of the concept of homotopy Independently, the similar notion was also developed by E.N. Dancer, K. Gęba and S. Rybicki. The main advantage of using otopies is that otopy relates maps with not necessarily the same domain.

Our main goal is to introduce such topology on the set of local maps in which otopies correspond to paths in this mapping space. Namely, we prove a version of the exponential law (adjoint theorem) which establishes the homeomorphism between the space of otopies and the space of paths of local maps. In fact, the above theorem is deduced from a more general result.

We use this results to explain and clarify the basic relations between different spaces of partial maps in an Euclidean space: local, proper, local gradient and proper gradient. We also pose some questions that arise naturally from presented results and observations. This is joint work with Piotr Nowak-Przygodzki.

Dynamics of a third-order rational difference equation

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In this paper, we study the global behavior of positive solutions of the difference equation

$$x_{n+1} = \frac{A + Bx_{n-1}}{C + Dx_n^p x_{n-2}^q},\ n = 0, 1, ...,$$

where the initial conditions x_0, x_{-1}, x_{-2} and the parameter B are non-negative real numbers, the parameters A, C, D are positive real numbers and p, q are fixed positive integers.

On the admissibility of set open topologies

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Let S be a topological monoid, X be a Tychonoff space and $C_\alpha(S,X)$ be the set of all continuous functions from S to X endowed with the set-open topology where α is a compact network on S. A topology τ on $C(S,X)$ is admissible if the evaluation mapping $e : S \times C(S,X) \to X$, defined by $e(s,f) = f(s)$, is continuous. We study the relationship between the admissibility of the set open topology on $C_\alpha(S,X)$ and the continuity of an action of S on $C(S,X)$. We prove that the set open topology on $C(S,X)$ is admissible if and only if $C_\alpha(S,X)$ with the action $s.f = (t \to f(st))$ is the cofree S-space over X in the category of Tychonoff spaces.

On a categorical unification of Stone duality, Priestley duality and the duality of spatial frames with sober topological spaces

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Motivated by the fixed-basis approaches to fuzzy topological spaces, for an abstract category **C**, a class $\mathcal{M}$ of **C**-monomorphisms and a fixed **C**-object L, **C**-$\mathcal{M}$-L-spaces and their category **C**-$\mathcal{M}$-L-**Top** were previously introduced by this author to provide a categorical framework for such approaches. Under some reasonable assumptions on **C**, the category **C** is dually adjoint to **C**-$\mathcal{M}$-L-**Top**. This adjunction is refined to a dual equivalence (the so-called Fundamental Categorical Duality Theorem, abbreviated as FCDT) between the full subcategory of **C** of all L-spatial objects and the full subcategory of **C**-$\mathcal{M}$-L-**Top** of all L-sober objects. FCDT is a categorical tool allowing us to put many existing and new dualities under the same umbrella. The aim of this study is to give the basic ingredients of FCDT, and is also to show how Stone duality, Priestley duality and the duality of spatial frames with sober topological spaces can be deduced from FCDT.

Uniform graded ditopology

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Graded ditopological texture spaces have been presented and discussed in categorical aspects by L.M. Brown and A. Šostak in [1]: Let $(S, \mathcal{S})$, $(V, \mathcal{V})$ be textures and consider $\mathcal{T}, \mathcal{K} : \mathcal{S} \to \mathcal{V}$ satisfying

1. $\mathcal{T}(S) = \mathcal{T}(\emptyset) = V$
2. $\mathcal{T}(A_1) \cap \mathcal{T}(A_2) \subseteq \mathcal{T}(A_1 \cap A_2)\ \forall A_1, A_2 \in \mathcal{S}$
3. $\bigcap_{j \in J} \mathcal{T}(A_j) \subseteq \mathcal{T}(\bigvee_{j \in J} A_j)\ \forall A_j \in \mathcal{S}, j \in J$
4. $\mathcal{K}(S) = \mathcal{K}(\emptyset) = V$
5. $\mathcal{K}(A_1) \cap \mathcal{K}(A_2) \subseteq \mathcal{K}(A_1 \cup A_2)\ \forall A_1, A_2 \in \mathcal{S}$
6. $\bigcap_{j \in J} \mathcal{K}(A_j) \subseteq \mathcal{K}(\bigcap_{j \in J} A_j)\ \forall A_j \in \mathcal{S}, j \in J$

Then the tuple $(S, \mathcal{S}, \mathcal{T}, \mathcal{K}, V, \mathcal{V})$ is called a graded ditopological texture space. In this work, considering [2], the authors generalize the structure of di-uniformity in ditopological texture spaces defined in [3] to the graded ditopological texture spaces and investigate graded ditopologies generated by graded di-uniformities.

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Some Characterizations of Mappings on Generalized Topological Spaces

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Generalized topological spaces are an important generalization of topological spaces. Let X be a set and $\mu \subseteq \exp X$. Then μ is called a generalized topology on X and (X, μ) is called a generalized topological space, if μ satisfies the following two properties:

1. $\emptyset \in \mu$.

2. Any union of elements of μ belongs to μ.

Min investigated (μ, ν)-continuous mappings from a generalized topological space (X, μ) to another generalized topological space (Y, ν), which were generalized continuous mappings introduced by Csaszar. He then obtained some interesting characterizations for (μ, ν)-continuous mappings in 2009.

We make further investigations for (μ, ν)-continuous mappings and develop some aspect of mapping theory in generalized topology. We consider some mappings which are defined on topological spaces for example open mappings, closed mappings, pseudo-open mappings, and quotient mappings, and we generalize them to the setting of generalized topological spaces. We obtain characterizations of these classes of mappings, and establish some relationships among these classes.

On a new Vietoris-type hypertopology

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In 1975, M. M. Choban ([1]) introduced a new topology on the set of all closed subsets of a topological space for obtaining a generalization of the famous Kolmogoroff Theorem on operations on sets. This new topology is similar to the upper Vietoris topology but is weaker than it. In 1998, G. Dimov and D. Vakarelov ([2]) used a generalized version of this new topology for proving an isomorphism theorem for the category of all Tarski consequence systems. Later on this generalized version was studied in details in [3]. In [4] we introduced a new lower-Vietoris-type hypertopology. Now we will introduce a Vietoris-type hypertopology and will study it.

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Notes on general filters in frames

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We look at the developments on convergence in pointfree topology spanning the work of Hong [1], the series of joint papers by Banaschewski and Hong [2], [3] and [4], Dube [5] and the recent paper by Dube and Naidoo [6]. We show our latest results in the theory of pointfree convergence and clustering with the concept of a general filter that provides new characterizations of (almost) compact and Boolean frames.

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Super Near Spaces

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In 1974, H. Herrlich [1] had introduced the concept of Nearness Space as an axiomatization of the concept of nearness of arbitrary collection of sets. Nearness is a fruitful concept in Topology because the fundamental aspects of unification and extension can be studied in a more general manner. Supertopological spaces defined by D. Doitchinov in 1964 [2], generalizes both topological and proximal spaces. In 2002 D. Lesberg [3] defined super nearness as the common concept of supertopology and nearness. Z.Vaziry and D. Leseberg [4] have introduced and studied completeness and completion in supernear spaces.

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On the topology of fuzzy strong b-metric spaces

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In this study, we introduce and investigate the concept of fuzzy strong b-metric spaces. By using the open balls, we define a topology on these spaces which is Hausdorff and first countable. Then we note that every separable fuzzy strong b-metric space is second countable. Moreover we give the uniform convergence theorem for these spaces.

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A Statistical Theory of Cluster Points

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The set of statistical (st) cluster points turn out to be very useful and interesting tool in turnpike theory to study optimal paths. It has also been discussed in convex or non-convex optimal control problems in discrete systems. In classical theory of convergence, statistical convergence and ideal convergence have a special place and these are also active research area. In this study, we investigate some inclusion relations for the set of st-cluster functions and the set of st-limit functions. We give examples and examine some of the properties of these sets. We also introduce the ideal convergence to a compact set using convergence theory to a set and a sequence of functions.

Partially topological gts-es

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The category **GTS** of generalized topological spaces (gts-es) and their strictly continuous mappings was introduced by H. Delfs and M. Knebusch in 1985. It implements the concept of Grothendieck topology in general topology. The full subcategory $\mathbf{GTS}_{pt}$ of partially topological gts-es was introduced in 2013. Both are topological constructs. A concrete reflector of smallification $sm : \mathbf{GTS}_{pt} \to \mathbf{SS}_{pt}$ and a concrete coreflector of topologization $top : \mathbf{GTS}_{pt} \to \mathbf{Top}$ distinguish two topological subconstructs of $\mathbf{GTS}_{pt}$. Additionally, $\mathbf{SS}_{pt}$ is closed under finite sums, while **Top** is hereditary. All topological theorems have their counterparts in $\mathbf{SS}_{pt}$ and many have also counterparts in $\mathbf{GTS}_{pt}$. For example, Tietze Extension Theorem can be formulated: *Let X be a normal partially topological space, and A its closed subset. For each strictly continuous function $f : A \to \mathbb{R}_{st}$ ($[0,1]_{st}$, resp.) there exists a strictly continuous extension $F : X \to \mathbb{R}_{st}$ ($[0,1]_{st}$, resp.).*

The subcategories: $\mathbf{LSS}_{pt}$ of locally small, partially topological gts-es and $\mathbf{WLS}_{pt}$ of weakly locally small ones are hereditary and finitely (co)productive. Here, beyond compactness, we have admissible compactness, reducing to smallness in $\mathbf{LSS}_{pt}$. Each topological group has its small, partially topological counterpart. Many have also locally small counterparts, e. g. $(\mathbb{R}^n_{lst}, +)$, $(\mathbb{Q}^n_{lst}, +)$. The theory of partially (para)topological groups is to be developed.

Categorifications of the polynomial ring

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In 2005 Helme–Guizon and Rong constructed a Khovanov-type homology theory whose Euler characteristics recovers the chromatic polynomial. Shumakovitch showed that the Khovanov homology of an alternating link contains only $\mathbb{Z}_2$ torsion. We adapt his proof to show that chromatic graph cohomology over algebra $\mathbb{Z}[x]/x^2$ also contains only $\mathbb{Z}_2$ torsion. Then we use a partial isomorphism between chromatic graph cohomology and Khovanov homology to show that semi-adequate links contain only $\mathbb{Z}_2$ torsion in certain gradings.

Gradient mapping of functions of several variables

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In 1950 Zygmunt Zahorski gave a complete description of level sets system corresponding to derivatives of real differentiable functions (see [1]). The problem of characterization of gradient mapping from $\mathbb{R}^n$, $n \geq 2$, remains open.

In this study, we focused on the question how "small" can preimages of open sets under the gradient mapping be with respect to Lebesgue measure and Hausdorff dimension.

[1] Z. Zahorski, *Sur la premiere derivee*, Trans. Amer. Math. Soc. **69** (1950), 1–54

[1] The study was supported by the Charles University in Prague, project GA UK No. 250001

On Hattori spaces

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For a subset A of the real line $\mathbb{R}$, Hattori space $H(A)$ is topological space whose underlying point set is the reals $\mathbb{R}$ and whose topology is defined as follows: points from A are given the usual Euclidean neighbourhoods while remaining points are given the neighbourhoods of the Sorgenfrey line $\mathbb{S}$. We give conditions on A which are sufficient and necessary for $H(A)$ to be respectively almost Cech-completeness, (quasi-)complete, Cech-analytic and weakly separated (in Tkachenko sense). We also prove that $H(A)$ is homeomorphic to $\mathbb{S}$ if (resp., only if) the closure of A in $\mathbb{S}$ is countable (resp., A is scattered). Some of these results solve questions raised by V.A. Chatyrko and Y. Hattori.

Semicontinuous functions and wQN-space

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In 2008, Scheepers' Conjecture about the relation of an $S_1(\Gamma,\Gamma)$-space and a wQN-space became an inspiration for L. Bukovský [1] to introduce notions of a wQN_*-space and a wQN^*-space. Both notions are modifications of a wQN-space using lower and upper semicontinuous functions. That was only a first step, the results related to these concepts justifying their introduction as well as modifications of other notions using semicontinuous functions just followed. We survey the applications of semicontinuous functions in singular sets related mainly to Scheepers' Conjecture and sequence selection principles.

[1] L. Bukovský, *On* wqn_* *and* wqn^* *spaces*, Topology and its Applications **156** (2008), no. 1, 24–27

[1] Supported by the grant 1/0097/16 of Slovak Grant Agency VEGA

On fiber strong shape Equivalences

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The purpose of this paper is the construction and investigation of fiber strong shape theory for compact metrizable spaces over a fixed base space $\mathbf{B_0}$, using the fiber versions of cotelescop, fibrant space and SSDR-map. In the paper obtained results containing the characterizations of fiber strong shape equivalences, based on the notion of double mapping cylinder over a fixed space $\mathbf{B_0}$. Besides, in the paper we construct and develop a fiber strong shape theory for arbitrary spaces over fixed metrizable space $\mathbf{B_0}$. Our approach is based on the method of Mardešić-Lisica and instead of resolutions, introduced by Mardešić, their fiber preserving analogues are used. The fiber strong shape theory yields the classification of spaces over $\mathbf{B_0}$ which is coarser than the classification of spaces over $\mathbf{B_0}$ induced by fiber homotopy theory, but is finer than the classification of spaces over $\mathbf{B_0}$ given by usual fiber shape theory.

[1] The authors supported by grant FR/233/5-103/14 from Shota Rustaveli National Science Foundation (SRNSF)

Finiteness of the discrete spectrum in a three-body system with point interaction

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In this poster we are concerned with a three-body system with point interaction, which is called the Ter-Martirosian–Skornyakov extension. We locate the bottom of the essential spectrum of that system and establish the finiteness of the discrete spectrum below the bottom. Our work here refines the result of R. A. Minlos ([1]), where the semi-boundedness of the operator is obtained.

[1] R. Minlos, *On point-like interaction between n fermions and another particle*, Mosc. Math. J. **11** (2011), no. 1, 113–127, 182

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TOPOSYM 2016 — BOOK OF ABSTRACTS

D. Chodounský, J. Verner (eds.)

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TOPOSYM is a series of international topology conferences held in Prague, Czech Republic in a five year cycle since 1961. Topics of the conference include general and set-theoretical topology, descriptive set theory, continuum theory, topological dynamics, topology in functional analysis, topology in algebra, and categorial topology.