Metrizable Cantor cubes that fail to be compact in some models for ZF

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The statement that every countable non-void collection of non-void finite sets has a choice function is denoted by CC(fin). It was proved by me in 2015 that it holds true in ZF that, for a set J, the Cantor cube 2^J is metrizable if and only if J is a countable union of finite sets. This implies that, in every model for ZF+¬CC(fin), there exist Cantor cubes that are simultaneously metrizable and not second-countable. I shall offer a proof to the following new result:

Theorem *It holds true in ZF that the following conditions (1)-(5) are all equivalent:*

1. *CC(fin).*
2. *Every metrizable Cantor cube is compact.*
3. *If a Cantor cube is metrizable, then its compact bornology is metrizable.*
4. *If a Cantor cube is first-countable, then its compact bornology is quasi-metrizable.*
5. *Every metrizable product of finite spaces is compact.*

Therefore, in every model for ZF+¬CC(fin), there exist metrizable Cantor cubes that are non-compact. Other consequences of it can be shown.

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