# Lindelöf spaces and large cardinals

*Toshimichi Usuba*[1]

usuba@waseda.jp

An important problem about Lindelöf spaces is the Arhangel'skii's problem. Arhangel'skii ([1]) asked if there is a Hausdorff Lindelöf space with points $G_\delta$ which has cardinality $> 2^{\aleph_0}$. Shelah ([2]) showed the consistency of the existence of such a space, but the consistency of the non-existence is still unknown. Scheepers and Tall ([3]) introduced indestructibly Lindelöf spaces, which can be characterized both by a topological game of length $\omega_1$ and by $\sigma$-closed forcing. Under a large cardinal assumption, they showed the consistency of no indestructibly Lindelöf space with points $G_\delta$ and of cardinality $> 2^{\aleph_0}$. Dias-Tall proved that the non-existence of such an indestructibly Lindelöf space is a large cardinal property. These results indicate some connection between large cardinals and Arhangel'skii's problem.

In this talk, we will present more connections between large cardinals and Lindelöf spaces with large cardinality but small pseudocharacter. We show that if $\square(\omega_2)$ holds, then there is a Lindelöf space with cardinality $\omega_2$ and points $G_\delta$. We also show that, under $V = L$, for each cardinal $\kappa$ there is a space with cardinality $\kappa^{++}$ but Lindelöf degree and pseudocharacter $\kappa$.

[1] A. Arhangel'skii, *On the cardinality of bicompacta satisfying the first axiom of countability*, Soviet Math. Dokl **10** (1969), no. 4, 951–955

[2] S. Shelah, On some problems in general topology, in *Set theory (Boise, ID, 1992–1994)* , number 192 in Contemp. Math. (Amer. Math. Soc., Providence, RI), p. 91–101.

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