The Bolzano property and the cube-like complexes

Przemysław Tkacz, Marian Turzański*

p.tkacz@uksw.edu.pl,
m.turzanski@uksw.edu.pl

B. Bolzano proved that if a function f continuous in a closed interval $[a, b]$ changes signs at the endpoints, i.e. $f(a) \cdot f(b) \leq 0$, then this function equals zero at one point of the interval at least. Nearly a hundred years later, H. Poincaré announced the n-dimensional version of this theorem. In 1940, C. Miranda rediscovered the Poincaré theorem and showed that it is equivalent to the Brouwer fixed point theorem.

During our talk we present a topological version of the Poincaré–Miranda theorem, called the Bolzano property. Next, we define the class of polyhedrons, called n-cube-like, which are generalization of the n-cubes. To show that they have the Bolzano property we adopt the n-dimensional version of Steinhaus chessboard theorem. Moreover, we investigate under what conditions the inverse limit preserves the Bolzano property and we give a characterization of the Bolzano property for locally connected spaces. Finally, we explain the relation between the Bolzano property and the covering dimension.