Cotorsion-free groups from a topological viewpoint

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We present a characterization of cotorsion-free Abelian groups in terms of homomorphisms from fundamental groups of Peano continua. For an open cover $\mathcal{U} \in Cov(X)$ of a space X, we consider the subgroup $\pi(\mathcal{U}, x)$ of $\pi_1(X, x)$, generated by all elements $[\alpha \cdot \beta \cdot \alpha^-]$ with $\beta \subseteq U \in \mathcal{U}$. We call a group G *homomorphically Hausdorff relative to* X if for every homomorphism $h : \pi_1(X, x) \to G$,

$$\bigcap_{\mathcal{U} \in Cov(X)} h(\pi(\mathcal{U}, x)) = 1.$$

We call G *Spanier-trivial relative to* X, provided

$$h(\bigcap_{\mathcal{U} \in Cov(X)} \pi(\mathcal{U}, x)) = 1.$$

Theorem *For an Abelian group G, the following are equivalent:*

1. *G is cotorsion-free.*
2. *G is homom. Hausdorff relative to every Peano continuum.*
3. *G is homom. Hausdorff relative to the Hawaiian Earring.*
4. *G is Spanier-trivial relative to the Griffiths twin cone.*

We also calculate the first homology group of the Griffiths twin cone.

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