Ideal quasi-normal convergence and related notions

Lev Bukovský, Pratulananda Das, Jaroslav Šupina*

bukovsky@kosice.upjs.sk,
pratulananda@yahoo.co.in,
jaroslav.supina@upjs.sk

Recently the presenter and D. Chandra began to study the notion of an ideal quasi-normal convergence and some topological notions defined by this convergence. We show that several properties of so introduced notions depend on the ideal and sometimes, they are also equivalent to some important property of the ideal. We present following results.

Theorem *The following are equivalent.*

1. $cof(\mathcal{J}) = \kappa$.
2. *For any set X and for any sequence of real functions* $f_n \to^{IQN} f$ *on* X, *there exit sets* X_ξ, $\xi < \kappa$ *such that* $X = \bigcup_{\xi<\kappa} X_\xi$ *and* $f_n \to^{\mathcal{J}-u} f$ *on each* X_ξ. *Moreover, if* X *is a topological space and* $f_n, n \in \omega$ *are continuous, then the sets* X_ξ *can be chosen to be closed.*

Theorem *The following are equivalent.*

1. *The set* C *is a pseudounion of the ideal* $\mathcal{J}$.
2. *For any set* X *and for any sequence of real functions* $f_n \to^{IQN} f$ *on* X, *with the control* $(\epsilon_n)_{n\in\omega}$ *there exist sets* $X_k, k \in \omega$ *such that* $X = \bigcup_k X_k$ *and* $f_n \to^{\langle C\rangle^* - \mathcal{J}} f$ *with same control on each* X_k.