On hyperstructures in topological categories

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We will propose and discuss a new approach to define hyperstructures, such as Vietoris hyperspaces in **Top**, which works in every cartesian closed topological category, and so applies to every topological category, using it's topological universe hull. The key tool is the embedding μ_X together with the projections π_A in the situation

$$C(X,Y) \xrightarrow{\mu_X} K(Y)^{K(X)} \cong \prod_{A \in K(X)} K(Y)_A \xrightarrow{\pi_A} (K(Y), \sigma_V)$$

as is explained in [1]:

Theorem *Let (Y, σ) be an infinite T_3-space. For every topological space let $C(X,Y)$ be equipped with compact-open topology. Let $\mathcal{B}$ be a class of topological spaces, that contains the Stone–Čech-compactification of a discrete space with cardinality at least card(Y).*

Then the Vietoris topology σ_V on $K(Y)$ is the final topology w.r.t. all $\pi_A \circ \mu_{(X,\tau)}$, $(X,\tau) \in \mathcal{B}$, $A \in K(X,\tau)$.

Using this as role model, we can examine appropriate hyperstructures for pseudotopological spaces, semiuniform convergence spaces or multifilter spaces, for example.

[1] R. Bartsch, *Vietoris hyperspaces as quotients of natural function spaces*, Rostock. Math. Kolloq. (2014/15), no. 69, 55–66

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